NAEG-ToE

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Please note: This is a draft; I currently work on it and will add new sections when time allows.

Abstract

The standard model (SM) of particle physics contains many arbitrary numerical constants probably not computable by means of the theory itself. In my opinion the SM has too few properties to allow their derivation. Therefore my approach to a theory of everything, besides other modifications, in principle is to replace the spinors of the SM by more complex objects. I replace them by a subset of special digraphs of 5 vertices and 5 edges, while the edges are interpreted as vectors living in an embedding space. These graphs' structures and dynamics lead to the observed behavior of nature, i.e. interactions, waves, constants, measured values etc..

In my 2014 publication - designated as first step of development - I worked out the structure and behavior of these graphs (actual fundamental particles, or field points) and roughly showed their association to elementary particles of the SM, but only gave an outlook to dynamics. Now I publish the second step of development, including dynamics, quantum mechanics and in this course expectedly the basis of general relativity. But despite this claim there are still many things to calculate and prove, and in addition to this to show that the SM of particle physics as well as general relativity is in fact being reproduced. Therefore this theory is still under development and modifications may be necessary while moving forward.

Beyond that I will also discuss my view on measurement, what actually would be the last step of development. But to possibly already show the whole theory, and since it's not much work, I will also write down these parts of the formulas.

Preface

My goal in engaging with theoretical physics is not only to find working formulas but rather sense the truth of nature. In this regard I am not yet completely satisfied with my current state - at some aspects it feels a bit unsleek from the philosophical point of view although until now math seems to be all right.

Despite I built up this theory from philosophical and logical consideration and deduction I will leave away all of that to save considerable time in writing. I will concentrate on hopefully as clear as possible mathematical formulations.

In this paper I go forward a big step from my 2014 publication. Besides eliminating some mistakes I introduce a completely new and better handling of the graphs, which are fundamental particles and "points" in the corresponding fields. For this I define many functions. Applying these to the dozens of distinct graphs they calculate some of the graphs' properties and generate matrices to work on them. In total that would mean one had to handle hundreds of matrices. That's inconvenient, actually impossible. Therefore a computer algebra system or a programming language should be used; I use Mathematica. Please refer to the corresponding files with implementations for all these functions.

Contents

Notation

- Sets and functions related to sets are noted as double stroke letters (e.g. distinct graphs: $\mathbb{G}$)
- Constants resp. functions generating them are noted as italic or calligraphic letters (e.g. the adjacency matrix for the p-th particle: $\mathcal{A}(p)$)
- Some general purpose functions, matrices and constants are noted as fracture or gothic style letters (e.g. 15-dimensional identity matrix: $\mathfrak{1}$, or the matrix to extract the g-th 3-vector from a 15-vector: $\mathfrak{i}(g)$)

References

In this paper several constants and measured values from SM are used. These were obtained from:

- "The NIST Reference on Constants, Units, and Uncertainty": physics.nist.gov/cuu/Constants
- "PDG particle data group": pdg.lbl.gov

Part I
The Theory

1 Introduction

Here I would like to provide a concise text based introduction to the theory. All this can be read off the formulas in section §2.

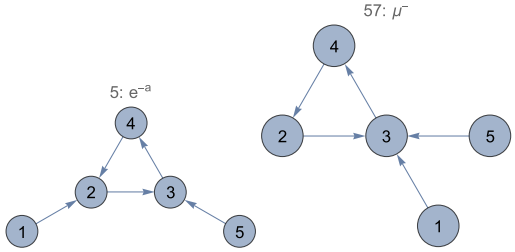
Constants This theory makes use of only a single unit-less constant $\ddot{u} \in \mathbb{N}$. It's exact numerical value is not yet known, but a reasonable value is denoted by $\ddot{U} = 1.58383 \times 10^{74}$.

Space and Time Our universe is embedded in a space describable by an euclidean $\mathbb{R}^n$. Thanks to the structure of the particles/graphs making up our universe, these only occupy 3 dimensions of that embedding space. Therefore it is possible to do calculations simply in $\mathbb{R}^3$. In contrast to that space our universe is space-like $\mathbb{N}^5$ (coordinates of the fields of the graphs); fundamental time (ticking of the universe) is also in some way discrete $\in \mathbb{N}$, but after every such a tick in part continuous running interactions take place; there time may be viewed as $\in \mathbb{R}$. The spatial dimensions $\mathbb{N}^5$ are closed/periodic for coordinates c, i.e. $f(\ldots c \ldots) = f(\ldots c + \ddot{u} \ldots)$.

Particles The basic objects in this theory are vertices G (3-vector) defining the start points of vectors, or viewed differently, the start points of directed edges of graphs. These edges' endpoints have to be at another G.

Fundamental particles are comprised of exactly 5 such objects and live in that just introduced temporal and spatial $\mathbb{R} \times \mathbb{R}^3$. So they are in fact a subset of directed graphs with 5 vertices and 5 edges (also denoted as G, but as 15-vectors). The vertices and their corresponding edges are numbered and ordered from 1 to 5 in every particle. All the edges of a single particle have the same maximum length $\in \mathbb{N}$; they are oscillating in length according to some rules after each tick of time of the universe.

If you imagine the embedding $\mathbb{R}^3$ with graphs something like the following two (there are in total probably exactly 66 thereof) in it, you are viewing a snapshot of one point of our universe. (Note that the triangles as a whole as well as the attached vectors may be oriented arbitrarily.)



Graphs that Define Particles Some rules apply for graphs to be a valid particle:

1. The graphs have to have a rotational degree of freedom of exactly 7 in $\mathbb{R}^3$. Graphs of that number of degrees of freedom necessarily have 5 vertices and 5 edges, and contain a triangle (the triangle has 3 and the remaining two edges each 2 degrees of rotational freedom.). Other graphs are incompatible to these ones and therefore may only exist in other universes.
2. All edge's maximum lengths have to be the same and be $\in \mathbb{N}$.
3. Edge's oscillations makes it necessary due to geometrical restrictions that the indices of the vertices of each graph's triangle build a sequence. ($\{1, 2, 3\}$, $\{2, 3, 4\}$ or $\{3, 4, 5\}$, no others possible)
4. Graphs having attached vertices at different triangle's vertices are restricted in maximum length to 1 due to the process of edges' length oscillation.

Two more rules apply but without strong rationale yet:

1. Maximum length of edges is 3. (Length > 3 would lead to very high energies/masses and electric charges higher than one elementary charge.)
2. It's not allowed that a vertex A is attached to the triangle and another vertex B is attached to vertex A, or in other words: two edges in a row attached to the triangle is forbidden.

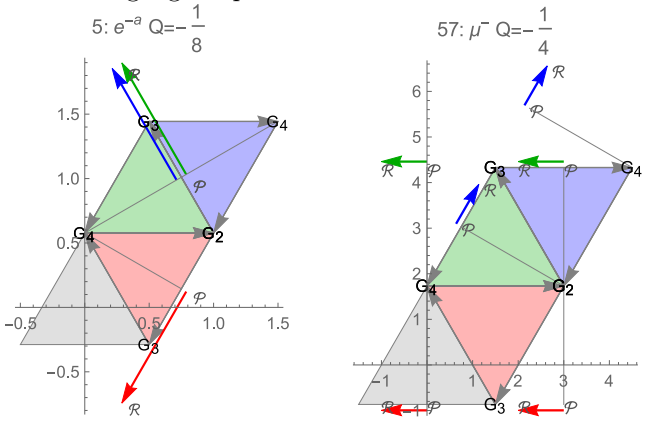
Applying these rules to all adjacency matrices and considering the edges' maximum lengths leads to 66 unique fundamental particles.

Intrinsic Dynamics and Interactions Particles are subject to intrinsic dynamics, which is the source of interactions/forces as well as the point of their effect; it's a good idea to view this dynamics as the emergence of particles and their properties, and not as a process actually taking place all the time, because this takes place between the ticks of universe's time. There are two types of dynamics: orientation and rotation.

Orientation Every vertex is reflected over it's single or two "parents" in the graph, one after another according to their indices. At the end of that process all the reflections are reversed in one step.

Rotation In the course of reflection the triangles' vertices which have no attached vertices undergo a 2π rotation. Fixed point resp. points is/are the one/ones of the triangle having attached vertices. Rotation direction is defined by the cross product of the directed edges.

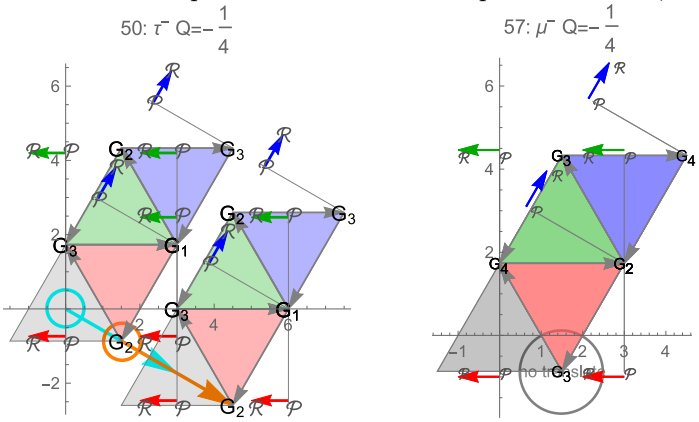
Resulting Behavior of Orientation and Rotation Leaving away the attached vertices and concentrating on the triangle gives pictures like these:



The gray triangle is the one before the first reflection takes place. Reflections are performed in order of the vertices'/Gs' indices to firstly form the red, then the green and finally the blue triangle. After every step of reflection, the continuous running rotations take place. Rotation is done around the point/s $\mathcal{P}$ and the vector/s $\mathcal{R}$. The left image (particle 5) shows one rotating vertex, G_4. The right image (particle 57) shows two rotating vertices, G_2 and G_4.

Oscillation Between the universe's ticks of time, also lengths of graphs' edges are subject to oscillation from their maximum lengths to 0 and back to their maximum lengths. During this process of contraction and expansion there can happen a translation due to a difference in expansion- and contraction-points. If there is translation taking place, this leads to "copies" of the graphs. These participate in inner particle interactions.

The left one of the following graphics shows an example of a such translated graph (At the end of the oscillation process the graph returns to it's initial coordinates.). Some particles are not subject to translation due to the rules for expansion and contraction points' location, as shown in the graphic on the right.



Rotation Space and Winding In this theory, time evolution is mainly the rotation of the graphs around their triangles' centers (the first, gray triangle's) in consequence of interactions induced by just introduced orientation and rotation dynamics. This happens implicitly by applying the result of interactions as a transformation of "rotation space".

The rotation space contains all three-dimensional orientations of the triangle as $\mathbb{S}^2 \times \mathbb{S}$. $\mathbb{S}^2$ contains all spherical orientations of the triangle's normal, and $\mathbb{S}$ contains all rotations around that normal.

Some particles have a twice as long "norm" of rotation space, which leads to a winding of rotation space of 2. Therefore there is another kind of "copies" (besides translation due to oscillation, as introduced above), namely two additional ones without transformation (for $\mathbb{S}^2$) and one rotated by $\pi/2$ around the triangle's normal and it's center (for $\mathbb{S}$).

Degrees of Freedom Every graph has 5 vertices, and each of them viewed without it's restrictions due to the graph's structure has 3 degrees of freedom (equal to the 3 dimensions of embedding space); that makes in total 15. But graphs' structures restrict them what leads to an important regrouping:

1. The triangle is determining to the graph (because of edges' directions). It has 9 degrees of freedom from which 3 are taken for the location of the whole graph.
2. Another 3 degrees of freedom go to the triangle's rotation. They build up the rotation space in which all orientations of the triangle can be found. Transforming this rotation space by interactions leads indirectly to rotation of the triangle. The size of every dimension of that rotation space is $1/h_t$. (h_t is the equivalent to Planck's constant.)
3. The remaining 3 degrees of freedom from the triangle's vertices are left free and build up the three dimensions of our world of experience. Their size is $1/h_f$. They get the coordinates $\{x, y, z\}$.
4. Both attached vertices undergo the same rules. They get 2 degrees of freedom for their rotation spaces; these spaces are rotated by interactions as well. Each dimension of these two rotation spaces have the size $1/h_a$.
5. Both attached vertices have each one degree of freedom left. These get the coordinates v resp. w. Their size is $1/h_f$ as well.

Taking that together particles form fields with five degrees of freedom $G(t, v, w, x, y, z)$. The coordinates {v,w,x,y,z} are subject to wave propagation.

2 Formulas

> In this part I will write down the theory just as it is. I will omit all explanations why it is that way, but concisely describe what the formulas do.

The correct formulation of the theory is done using discrete space ($\{v, w, x, y, z\} \in \mathbb{N}$) and time ($t \in \mathbb{N}$) for the particle's fields but continuous coordinates $\in \mathbb{R}$ make some calculations simpler. The differences in the formulas in these two cases are not big; therefore both are written down at once.

All particle functions obey periodicity individually for every spatial coordinate c like $\mathbb{F}_p(t, \dots c + \ddot{u} \dots) = \mathbb{F}_p(t, \dots c \dots)$.

To treat all the different graphs in a transparent way, at this point all the function from section §12 are introduced. Please refer to that section.

2.1 Equation of Motion

The equation of motion reads:

$$\begin{cases} \Delta\mathbb{F}_p = w_p + c\Delta_p + i\Delta_p & \text{discrete} \\ \underbrace{\partial_t \mathbb{F}_p}_{1} = \underbrace{w_p}_{2} + \underbrace{c\delta_p}_{3} + \underbrace{i\delta_p}_{4} & \text{continuous} \end{cases} \tag{1}$$

$\mathbb{F}$ is the list of particle fields, each being specified by a 15-vector, p runs from 1 to $\mathcal{C}$. Thus this is a system of $15\mathcal{C} = 990$ equations.

There is no superposition-principle to be applied; these equations are simply a description of what's going on in nature, not a representation of nature. Waves of different form and/or length exist in the "parallel spaces" with coordinates x, y, z but different v and/or w.

1. This defines the change in time of all the particle-fields.

   disc. $\Delta\mathbb{F}$ stands for the change when the universe undergoes one time tick, e.g. for particle number 5 and the z-coordinate of it's first vertex, which is denoted as c: $\Delta G5c(t, \dots) = G5c(t+1, \dots) - G5c(t, \dots)$

   cont. Since $\Delta\mathbb{F}$ is the change from t to $t+1$, in the continuous case simply the partial derivative in respect to time without any factor has to be used.

2. This is the wave term of the fields, thus bears the spatial connection resp. spatial partial derivatives. This term leads to propagation both in the embedding and the embedded space by transformation comprised of translation and rotation of the graphs. There's no connection between different particle fields in it. See section 2.2

3. The wave term induces transformations. Since transformations are the reason for interactions these transformations lead to wave caused interactions. See section 2.3

4. This is the interaction term describing interactions both between different particles as well as within a single particle.

   While translations are the same in discrete and continuous cases, there's a difference in regard of rotations:

   disc. The rotations are returned as the difference between the current and the new orientation; these are computed by means of the rotation around an axis using $\mathfrak{R}()$.

   cont. The generators of the rotations are returned.

   See section 2.4

2.2 Wave Term w

The wave term w in principle exists only because particles are forced to take on the position and orientation of it's neighbors in regard of the field coordinates. This transformation is composed of a translation of the whole graph as well as a rotation of its edges. It's useful to implement these in separate functions:

$$w_p = \underbrace{wg(p)}_{1} + \underbrace{wt(p)}_{2} + \underbrace{wp(p)}_{3} + \underbrace{wa(p)}_{4} \tag{2}$$

w_p is a 15-vector.

1. This is the term for translation of the whole graph. See section 2.2.1

2. This is the triangle's rotation. See section 2.2.2

3. When rotating the triangle, attached vertices have to follow this rotation by a translation. See section 2.2.3

4. Rotation of vertices attached to the triangle. See section 2.2.4

Hereafter applies:

$$d\mathbb{F}_{p,c} = \underbrace{\begin{cases} \mathbb{F}_p(c+1) - \mathbb{F}_p(c) & \text{disc.} \\ \partial_c \mathbb{F}_p & \text{cont.} \end{cases}}_{1}$$

1. This is the difference between neighboring field points in discrete space resp. the spatial derivative as

   disc. the difference between the field points in direction of the dimension according to coordinate c or

   cont. the partial derivative of $\mathbb{F}$ in direction of coordinate c.

2.2.1 Wave Induced Graph Translation wg

$$wg(p) = \underbrace{\mathfrak{A}\cdot}_{1} \left(\underbrace{\underbrace{\sum_{A}^{2}}_{a} \underbrace{\mathfrak{i}(\mathbb{A}(p)_A) \cdot d\mathbb{F}_{p,\mathbb{C}_A}}_{b}}_{2} + \underbrace{\underbrace{\sum_{c}^{\{x,y,z\}}}_{a} \underbrace{\sum_{A}^{\mathbb{T}(p)} \frac{1}{3}}_{b} \underbrace{\mathfrak{i}(A) \cdot d\mathbb{F}_{p,c}}_{c}}_{3} \right) \tag{3}$$

$wg(p)$ returns a 15-vector.

1. The terms numbered with 2 (attached vertices) and 3 (triangle vertices) both contribute to translation and give 3-vectors. By multiplying these translations with $\mathfrak{A}$, they can be added to the graph's 15-vector.

2. Waves affecting vertices attached to the triangle.

(a) All to the triangle attached vertices contribute, each in it's own dimension, but here summation is not done over the vertex index but over their number.

(b) $\mathbb{A}(p)_A$ returns the vertex index of the A-th attached vertex whose coordinate is selected by $\mathfrak{i}()$ from $d\mathbb{F}_{p,\mathbb{C}_A}$.

3. Waves affecting the triangles' vertices.

(a) Dimensions $\{x, y, z\}$ are the triangle's spatial degrees freedom; waves run in all these dimensions, thus they have to be summed up.

(b) The transformation takes place for all the vertices A of the triangle determined by $\mathbb{T}(p)$. But only dimensions have to be added up, not vertices, since they form the triangle. So, the mean is calculated by multiplication with 1/3.

(c) $\mathfrak{i}(A)$ selects the A-th 3-vector from $d\mathbb{F}_{p,c}$.

2.2.2 Wave Induced Triangle Rotation wt

$$v = \underbrace{\sum_{c}^{\{x,y,z\}}}_{1} \underbrace{\mathfrak{p}\left(\underbrace{\frac{\sqrt{3}}{\mathcal{L}(p)}}_{a} \underbrace{\sum_{A}^{\mathbb{T}(p)} \frac{1}{3}}_{b} \underbrace{((\mathfrak{i}(A) - \mathcal{X}(p)) \cdot \mathbb{F}_p) \times ((\mathfrak{i}(A) - \mathcal{X}(p)) \cdot d\mathbb{F}_{p,c})}_{c}, \underbrace{\mathcal{X}(p) \cdot d\mathbb{F}_{p,c}}_{d}\right)}_{2}$$

$$wt(p) = \underbrace{\begin{cases} \mathcal{M}(p, v, 1, 0) \cdot \mathbb{F}_p & \text{disc.} \\ v \cdot \mathcal{I}(p, 1, 0) \cdot \mathbb{F}_p & \text{cont.} \end{cases}}_{3} \tag{4}$$

$wt(p)$ returns a 15-vector.

1. All the triangle's dimensions have to be added up to form the rotation.

2. The rotation of the particle is defined by a rotation vector v. This vector is the sum of rotation vectors being parallel (never anti-parallel) to the coordinate axes. This is made sure by $\mathfrak{p}(v, n)$.

(a) This is just a purely mathematical normalization factor.

(b) To calculate v, the mean of the rotation vectors for all three triangle vertices is calculated.

(c) The latter rotation vectors are the cross product of the vertices' positions and their rotation generator.

(d) Coordinate axes in embedding space equal the direction of the triangle centers $\mathcal{X}(p)$ change when following the coordinates in embedded space.

3. The rotation vector v is applied to the graph's triangle like for interactions.

2.2.3 Wave Induced Triangle Rotation Correction wp

Length of graph's edges does not change. Thus if rotating a graph's triangle, attached vertices have to move along that rotation. This is represented by the function $wp()$.

$$wp(p) = \underbrace{\sum_{A}^{\mathbb{A}(p)}}_{1} \underbrace{\mathfrak{I}(A)}_{2} \cdot \underbrace{\mathfrak{i}(\mathbb{D}(p, A))}_{3} \cdot \underbrace{wt(p)}_{4} \tag{5}$$

$wp(p)$ returns a 15-vector.

1. Translate the position of all attached vertices $\mathbb{A}(p)$.

2. The third part of the formula returns a 3-vector; here, this is written into the A-th vertex of the graph.

3. The translation is read from the vertices' parent vertices $\mathbb{D}(p, A)$.

4. The translation amount equals the rotation $wt(p)$ of the corresponding triangle vertex.

2.2.4 Wave Induced Rotation of Attached Vertices wa

$$v(c) = \underbrace{\mathfrak{p}\left(\underbrace{\frac{1}{\mathcal{L}(p)}}_{a}\underbrace{(\mathfrak{i}(\mathbb{A}(p)_c)\cdot\mathcal{V}(p)\cdot\mathbb{F}_p)\times(\mathfrak{i}(\mathbb{A}(p)_c)\cdot\mathcal{V}(p)\cdot d\mathbb{F}_{p,c})}_{b},\underbrace{\mathfrak{i}(\mathbb{D}(p,\mathbb{A}(p)_c))\cdot d\mathbb{F}_{p,c}}_{c}\right)}_{1}$$

$$wa(p) = \underbrace{\sum_{c}^{2}}_{2}\underbrace{\mathfrak{D}(\mathbb{A}(p)_c)}_{3}\cdot\underbrace{\begin{cases}\mathcal{M}(p,v(c),0,1)\cdot\mathbb{F}_p & \text{disc.}\\ v(c)\cdot\mathcal{I}(p,0,1)\cdot\mathbb{F}_p & \text{cont.}\end{cases}}_{4} \tag{6}$$

$wa(p)$ returns a 15-vector.

1. The rotations of attached vertices are defined by rotation vectors $v(c)$. These vectors are parallel (never anti-parallel) to the coordinate axes. This is made sure by $\mathfrak{p}(v,n)$.
   (a) This is just a purely mathematical normalization factor.
   (b) Each of the rotation vectors is based on the cross product of the vertices' positions and their rotation generator.
   (c) Coordinate axes in embedding space equal the direction of the position change of the parent vertices of the vertices to be rotated when following the coordinates in embedded space.
2. Both attached vertices are rotated. Summation runs over their coordinate index; the actual vertex index is queried by means of $\mathbb{A}(p)_c$.
3. Takes only the corresponding coordinate values for vertex $\mathbb{A}(p)_c$ from the rest of the formula.
4. The rotation vector $v(c)$ is applied to the vertices attached to the triangle like for interactions.

2.3 Connection $c\Delta$ resp. $c\delta$

Particles always interact with all the others, but to do some particular calculations it's useful to also define functions that calculate the interactions between a given set of particles. Therefore here two sets of functions will be defined despite in this chapter only the functions for all particles' interactions play a role.

Convenience Functions wT and wR

In both cases there are two functions that contribute as sources of wave induced translation and rotation. These functions are combined to each form single functions:

$$\begin{cases} wT(Gs,dGs) = wg(Gs,dGs) + wp(Gs,dGs) & \text{all particles}\\ wT(ps,Gs,dGs) = wg(ps,Gs,dGs) + wp(ps,Gs,dGs) & \text{particle numbers ps}\end{cases} \tag{7}$$

$$\begin{cases} wR(Gs,dGs) = wt(Gs,dGs) + wa(Gs,dGs) & \text{all particles}\\ wR(ps,Gs,dGs) = wt(ps,Gs,dGs) + wa(ps,Gs,dGs) & \text{particle numbers ps}\end{cases} \tag{8}$$

Hereafter $wT()$ is used for function parameters tGs and $wR()$ for rGs.

For all Particles

$$\begin{cases} c\Delta(Gs,tGs,rGs)_{pB} = m(pB, c(Gs,tGs,rGs,True)_{pB})\cdot Gs_{pB} + \mathfrak{A}\cdot c(Gs,tGs,rGs,True)_{pB} - \underbrace{Gs_{pB}}_{3} & \text{disc.}\\ c\delta(Gs,tGs,rGs)_{pB} = \underbrace{c(Gs,tGs,rGs,False)_{pB}\cdot i(pB)\cdot Gs_{pB}}_{1} + \underbrace{\mathfrak{A}\cdot c(Gs,tGs,rGs,False)_{pB}}_{2} & \text{cont.}\end{cases} \tag{9}$$

Gs is the list of all graph/vertex coordinates of all particles, e.g. $\mathbb{F}_{pB}$ or $\mathcal{G}(pB)$; $pB = 1\ldots\mathcal{C}$.

1. The first term of the connection function defines the rotation of the particle due to wave induced interactions. In the discrete case it is a rotation (performed by the rotation matrix m, applied to Gs) around the 3-vector returned by $c()$ and the rotation angle is equal to it's length. In the continuous case there's no big difference. i returns a 3-vector of 15×15-matrices which allows to calculate the rotation generators analogous to m.

2. The second term is a translation according to the connection 3-vector. To apply this to all 5 vertices of the graphs it has to be multiplied by $\mathfrak{A}$.

3. Gs_{pB} has to be subtracted in the discrete case since $\mathcal{M}$ is the transformation matrix to calculate the rotated graph, not the difference from Gs_{pB}.

For a Specified Set of Particles

In contrast to the functions for all particles, here a parameter is added representing the set of particle numbers to operate on. Note that PB is not a particle number like pB; it's an index in the set of particles ps. Besides this there's no difference to above.

$$\begin{cases} c\Delta(ps, Gs, tGs, rGs)_{PB} = & \\ m(ps_{PB}, c(ps, Gs, tGs, rGs, True)_{PB}) \cdot Gs_{PB} + \mathfrak{A} \cdot c(ps, Gs, tGs, rGs, True)_{PB} - Gs_{PB} & \text{disc.} \\ c\delta(ps, Gs, tGs, rGs)_{PB} = & \\ c(ps, Gs, tGs, rGs, False)_{PB} \cdot i(ps_{PB}) \cdot Gs_{PB} + \mathfrak{A} \cdot c(ps, Gs, tGs, rGs, False)_{PB} & \text{cont.} \end{cases} \tag{10}$$

2.3.1 Connection Function c

This function describes the connection effect between all respectively a set of particles. It's just the sum of orientation and rotation connection. There's no difference between discrete and continuous calculation.

$$c(Gs, tGs, rGs, disc) = cO(Gs, tGs) + cR(Gs, rGs, disc) \tag{11}$$

$$c(ps, Gs, tGs, rGs, disc) = cO(ps, Gs, tGs) + cR(ps, Gs, rGs, disc) \tag{12}$$

Orientation Connection cO This function defines the effect of orientation connection as a result of translations due to wave propagation.

$$co(\underbrace{AM}_{1}, \underbrace{BM}_{2}, \underbrace{AR}_{3}) = \underbrace{\mathfrak{N}(BM - AM)\, AR}_{4}$$

1. Location of source point of the effect in particle pA.

2. Coordinates of the reflection center in destination particle pB.

3. Translation vector from source particle pA.

4. Result is a 3-vector.

$$cO(Gs, tGs)_{pB} = \underbrace{\sum_{B}^{5}}_{1} \underbrace{\sum_{pA}^{\mathcal{C}} \sum_{A}^{5}}_{2} \underbrace{\begin{cases} co(AM, BM, AR) & pA \neq pB \vee A \neq B \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & \text{else} \end{cases}}_{3} \tag{13}$$

Parameters of $co()$ are as follows:

$$\begin{aligned} AM &= \mathfrak{i}(A) \cdot Gs_{pA} \\ BM &= M(pB, B, B, Gs_{pB}) \\ AR &= \mathfrak{i}(A) \cdot tGs_{pA} \end{aligned}$$

$cO()$ returns a 3-vector.

1. Orientation connection operates on every particle pB's orientation states B equaling the reflections of vertex B, it's translation states bT and winding states bW.

2. The sources of orientation connection are all wave induced translations of vertices of particles pA.

3. A vertex has no effect on itself.

Definition of function $cO(ps, Gs)$ is almost identical; it simply restricts the connection effect to the particle numbers ps, and Gs contains only the corresponding graph coordinates.

Rotation Connection cR Rotation connection effects are defined by a rotation vector as the axis around which the rotation takes place; it's length is the rotation angle. Since rotation interactions due to waves are not rotations about 2π the correct angle has to be obtained from the transformation induced by the wave. Since in continuous case the transformation is the rotation generator one can simply take the cross product between that generator and the normalized vector from the point of rotation towards the rotating vertex. In the case of discrete time evolution the transformation is only approximately the rotation generator (only at the limit of 0 energy) and is not perpendicular to the vector from the point of rotation to the rotating vertex. This discrepancy has to be corrected using following function:

$$cRV(L,P,R) = P \times R \begin{cases} 2\frac{\arcsin(\frac{\mathfrak{N}(R)}{2L})}{\mathfrak{N}(P \times R)} & \text{disc.} \\ \frac{1}{L} & \text{cont.} \end{cases}$$

L is the length of P. P is the vector from the point of rotation towards the rotating vertex. R is the transformation resp. rotation generator.

This function defines the effect of rotation connection as a result of rotations due to wave propagation.

$$cr(\underbrace{AP}_{1}, \underbrace{BP}_{2}, \underbrace{AR}_{3}, \underbrace{BR}_{4}, \underbrace{BV}_{5}) = \underbrace{\mathfrak{N}(BP - AP)\, AR \cdot BR\, BV}_{6}$$

1. Location of source point of the effect in particle pA.
2. Coordinates of the rotation center in destination particle pB.
3. Rotation vector in particle pA.
4. Rotation vector in particle pB.
5. Vector around which particle pB rotates in time evolution - $\mathcal{D}(pB) \cdot \mathbb{F}_{pB}$.
6. Result is a 3-vector.

$$cR(Gs, rGs, disc)_{pB} = \underbrace{\sum_{B}^{\mathbb{R}(pB)} \sum_{b}^{\mathbb{O}(pB)}}_{1} \underbrace{\sum_{pA}^{\mathcal{C}}}_{2} ($$
$$\underbrace{\sum_{A}^{\mathbb{T}(pA)} \begin{cases} cr(APt, BP, ARt, BR, BV) & pA \neq pB \vee A \neq B \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & \text{else} \end{cases}}_{3} +$$
$$\underbrace{\sum_{A}^{\mathbb{A}(pA)} \begin{cases} cr(APa, BP, ARa, BR, BV) & pA \neq pB \vee A \neq B \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & \text{else} \end{cases}}_{4}) \quad (14)$$

Parameters of $cr()$ are as follows:

$$\begin{aligned} APt &= \mathcal{X}(pA) \cdot Gs_{pA} \\ ARt &= cRV(\frac{\mathcal{L}(pA)}{\sqrt{3}}, (\mathfrak{i}(A) - \mathcal{X}(pA)) \cdot Gs_{pA}, \mathfrak{i}(A) \cdot rGs_{pA}) \end{aligned}$$

$$\begin{aligned} APa &= \mathfrak{i}(\mathbb{D}(pA, A)) \cdot Gs_{pA} \\ ARa &= cRV(\mathcal{L}(pA), (\mathfrak{i}(A) - \mathfrak{i}(\mathbb{D}(pA, A))) \cdot Gs_{pA}, \mathfrak{i}(A) \cdot rGs_{pA}) \end{aligned}$$

$$\begin{aligned} BP &= P(pB, B, b, Gs_{pB}) \\ BR &= R(pB, B, b, Gs_{pB}) \\ BV &= \mathcal{D}(pB) \cdot Gs_{pB} \end{aligned}$$

$cR()$ returns a 3-vector.

1. The connection has an effect on all rotating vertices B in all their orientation states b corresponding to the triangle's.
2. Every particle pA is the source of that effect.
3. The triangle's rotation is a source; vertices don't interact with themselves.
4. The rotation of vertices attached to the triangle are also a source; vertices don't interact with themselves.

Definition of function $cR(ps, Gs)$ is almost identical; it simply restricts the connection effect to the particle numbers ps, and Gs contains only the corresponding graph coordinates.

2.4 Interaction $i\Delta$ resp. $i\delta$

Particles always interact with all the others, but to do some particular calculations it's useful to also define functions that calculate the interactions between a given set of particles. Therefore here two sets of functions will be defined despite in this chapter only the functions for all particles' interactions play a role.

For all Particles

$$\begin{cases} i\Delta(Gs)_{pB} = \mathcal{M}(pB, i(Gs)_{pB}) \cdot Gs_{pB} + \frac{h_f \mathfrak{A} \cdot i(Gs)_{pB}}{d(pB)\mathcal{N}(pB)} - \underbrace{Gs_{pB}}_{3} & \text{disc.} \\ i\delta(Gs)_{pB} = \underbrace{i(Gs)_{pB} \cdot \mathcal{I}(pB) \cdot Gs_{pB}}_{1} + \underbrace{\frac{h_f \mathfrak{A} \cdot i(Gs)_{pB}}{d(pB)\mathcal{N}(pB)}}_{2} & \text{cont.} \end{cases} \tag{15}$$

Gs is the list of all graph/vertex coordinates of all particles, e.g. $\mathbb{F}_{pB}$ or $\mathcal{G}(pB)$; $pB = 1 \ldots \mathcal{C}$.

1. The first term of the interaction function defines the rotation of the particle due to interaction. In the discrete case it is a rotation (performed by the rotation matrix $\mathcal{M}$, applied to Gs) around the 3-vector returned by $i()$ and the rotation angle is equal to it's length. In the continuous case there's no big difference. $\mathcal{I}$ returns a 3-vector of 15×15-matrices which allows to calculate the rotation generators analog to $\mathcal{M}$. In both cases rotation speed is defined by the size of the space rotations operate within; rotations of the triangle thus are subject to the factor h_t and the attached vertices are subjected to h_a - both included in $\mathcal{M}$ resp. $\mathcal{I}$.
2. The second term is a translation according to the interaction 3-vector. To apply this to all 5 vertices of the graphs it has to be multiplied by $\mathfrak{A}$. Here, the rotation space also has it's own size, taken into account by h_f.
3. Gs_{pB} has to be subtracted in the discrete case since $\mathcal{M}$ is the transformation matrix to calculate the rotated graph, not the difference from Gs_{pB}.

For a Specified Set of Particles

In contrast to the functions for all particles, here a parameter is added representing the set of particle numbers to operate on. Note that PB is not a particle number like pB; it's an index in the set of particles ps. Besides this there's no difference to above.

$$\begin{cases} i\Delta(ps, Gs)_{PB} = \mathcal{M}(ps_{PB}, i(ps, Gs)_{PB}) \cdot Gs_{PB} + \frac{h_f \mathfrak{A} \cdot i(ps, Gs)_{PB}}{d(ps_{PB})\mathcal{N}(ps_{PB})} - Gs_{PB} & \text{disc.} \\ i\delta(ps, Gs)_{PB} = i(ps, Gs)_{PB} \cdot \mathcal{I}(ps_{PB}) \cdot Gs_{PB} + \frac{h_f \mathfrak{A} \cdot i(ps, Gs)_{PB}}{d(ps_{PB})\mathcal{N}(ps_{PB})} & \text{cont.} \end{cases} \tag{16}$$

2.4.1 Interaction Function i

This function describes interaction between all respectively a set of particles. It's just the sum of orientation and rotation interactions. There's no difference between discrete and continuous calculation.

$$i(Gs) = iO(Gs) + iR(Gs) \tag{17}$$

$$i(ps, Gs) = iO(ps, Gs) + iR(ps, Gs) \tag{18}$$

Orientation Interaction iO This function defines the effect of orientation interaction. (The following function io is extracted from iO because for some calculations it's useful to use other implementations.)

$$io(\underbrace{AX}_{1},\underbrace{BX}_{2},\underbrace{AM}_{3},\underbrace{BM}_{4},\underbrace{AR}_{5}) = \underbrace{\mathfrak{N}(BM - AM)\,AR}_{6}$$

1. Triangle center $\mathcal{X}(p)$ of source particle pA.
2. Triangle center $\mathcal{X}(p)$ of destination particle pB.
3. Reflection center of source particle pA.
4. Reflection center of destination particle pB.
5. Reflection vector from source particle pA.
6. Result is a 3-vector.

$$iO(Gs)_{pB} = \underbrace{d(pB)\mathcal{N}(pB)}_{1} \underbrace{\sum_{bW}^{w(pB)} \sum_{bT}^{t(pB)} \sum_{B}^{5}}_{2} \underbrace{\sum_{pA}^{\mathcal{C}} \sum_{aW}^{w(pA)} \sum_{aT}^{t(pA)} \sum_{A}^{5}}_{3}$$
$$\underbrace{\begin{cases} io(AX, BX, AM, BM, AR) & pA = pB \wedge A \neq B \vee pA \neq pB \wedge aT = bT = 1 \wedge aW = bW = 1 \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & \text{else} \end{cases}}_{4} \tag{19}$$

Parameters of $io()$ are as follows:

$$\begin{aligned} AX &= \mathcal{X}(pA) \cdot Gs_{pA} \\ BX &= \mathcal{X}(pB) \cdot Gs_{pBA} \\ AM &= M(pA, aW, aT, A, A, Gs_{pA}) \\ BM &= M(pB, bW, bT, B, B, Gs_{pBA}) \\ AR &= M(pA, aW, aT, A, A, Gs_{pA}) - G(pA, aW, aT, A, A-1, Gs_{pA}) \end{aligned}$$

$iO()$ returns a 3-vector.

1. Just because of convenience, the norm of rotation space is included here instead of in $i\Delta()$.
2. Orientation interaction operates on every particle pB's orientation states B equaling the reflections of vertex B, it's translation states bT and winding states bW.
3. The sources of orientation interaction are all particles pA, all their vertices' reflections A, their translation states aT and winding states aW.
4. A vertex has no effect on itself, and between different graphs vertices affect each other only in the first translation and winding state.

Definition of function $iO(ps, Gs)$ is almost identical; it simply restricts the interaction to the particle numbers ps, and Gs contains only the corresponding graph coordinates.

Rotation Interaction iR This function defines the effect of rotation interaction. (The following functions irs and irf are extracted from iR because for some calculations it's useful to use other implementations.)

$$irs(\underbrace{AX}_{1},\underbrace{BX}_{2},\underbrace{AP}_{3},\underbrace{BP}_{4},\underbrace{AR}_{5},\underbrace{BR}_{6},\underbrace{BV}_{7}) = \underbrace{\exp(\mathfrak{N}(BP - AP)\,AR \cdot BR)\,BV}_{8}$$

$$irf(\underbrace{AX}_{1},\underbrace{BX}_{2},\underbrace{AP}_{3},\underbrace{BP}_{4},\underbrace{AR}_{5},\underbrace{BR}_{6},\underbrace{BV}_{7}) = \underbrace{\mathfrak{N}(BP - AP)\,AR \cdot BR\,BV}_{8}$$

1. Triangle center $\mathcal{X}(p)$ of source particle pA.
2. Triangle center $\mathcal{X}(p)$ of destination particle pB.

3. Rotation center of source particle pA.
4. Rotation center of destination particle pB.
5. Rotation vector of source particle pA.
6. Rotation vector of destination particle pB.
7. Vector around which particle pB rotates in time evolution - $\mathcal{D}(pB) \cdot \mathbb{F}_{pB}$.
8. Result is a 3-vector.

$$iR(Gs)_{pB} = \underbrace{d(pB)\mathcal{N}(pB)}_{1} \underbrace{2\pi}_{2} \underbrace{\sum_{bW}^{w(pB)} \sum_{bT}^{t(pB)} \sum_{B}^{\mathbb{R}(pB)} \sum_{b}^{\mathbb{O}(pB)}}_{3} \underbrace{\sum_{pA}^{\mathcal{C}} \sum_{aW}^{w(pA)} \sum_{aT}^{t(pA)} \sum_{A}^{\mathbb{R}(pA)} \sum_{a}^{\mathbb{O}(pA)}}_{4}$$
$$\underbrace{\begin{cases} irs(AX, BX, AP, BP, AR, BR, BV) & pA = pB \wedge A \neq B \\ irf(AX, BX, AP, BP, AR, BR, BV) & pA \neq pB \wedge aT = bT = 1 \wedge aW = bW = 1 \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & \text{else} \end{cases}}_{5} \tag{20}$$

Parameters of $irs()$ resp. $irf()$ are as follows:

$$\begin{aligned} AX &= \mathcal{X}(pA) \cdot Gs_{pA} \\ BX &= \mathcal{X}(pB) \cdot Gs_{pB} \\ AP &= P(pA, aW, aT, A, a, Gs_{pA}) \\ BP &= P(pB, bW, bT, B, b, Gs_{pB}) \\ AR &= R(pA, aW, A, a, Gs_{pA}) \\ BR &= R(pB, bW, B, b, Gs_{pB}) \\ BV &= \mathcal{D}(pB) \cdot Gs_{pB} \end{aligned}$$

$iR()$ returns a 3-vector.

1. Just because of convenience, the norm of rotation space is included here instead of in $i\Delta()$.
2. Interaction is calculated using normalized rotation vectors representing rotations about 2π, so a multiplication by 2π has to be added here.
3. Rotation interaction operates on every particle pB's orientation states b, rotating vertex B, translation state bT and winding state bW.
4. The sources of rotation interaction are all particles pA, all their orientation states a of their rotating vertices A, their translation states aT and winding states aW.
5. Interaction within a single particle is, except self-interaction of a vertex, the function $irs()$; interaction with vertices of other particles is the function $irf()$. Only self-interaction of a vertex leads to no effect.

Part II
Discussion

I decided to split this section into three parts: First I introduce general contents, which cover local as well as global properties of this theory, and can't be discussed solely in one of the following sections. Secondly interactions taking place at a local point of the space of our experience (embedded space) are considered, while in fact in this theory this "point" is a space as well (in embedding space). That means in second section I will not take a look at waves. These will be treated in the third section.

Please note that all calculations in this part only apply to idealized states, because in particular, here the asymmetric particles of this theory are viewed as symmetric ones, perturbations from dimensions v and w are ignored, and limits are used as approximations. These things can lead to discrepancies from known or measured values, of course.

3 General

3.1 Mass Contributions

This theory fully explains the cause of rest mass without using any constants, only by applying rules to the graphs. These rules make the appearance of mass a calculation that can't be put in a short formula. Mass is composed of:

1. Orientation and rotation interactions within a single fundamental particle: In fact, this is the main contribution of mass for almost all of the particles.
2. Rotation interaction of a fundamental particle with it's antiparticle: This is a little part, even for electron only a correction.
   At this point, calculation of muon and tauon mass is already possible.
3. Orientation and rotation interaction between fundamental particles residing at the same point in embedding space: This applies for all composite particles and is only a small contribution.
4. Rotation interaction of a fundamental particle with the antiparticles of the fundamental particles that build with it the composite particle: Similar to (2) this is only a correction.
   Up to here, everything can be calculated at just one coordinate of embedded space, and thus is treated in section §4.
   All not light composite particles bear an additional mass. But electron's and positron's mass can already be calculated reasonably accurately at this point.
5. Lastly, the just mentioned additional mass mainly arises from electromagnetism in connection with the effects of confinement.
   This calculation needs wave propagation, of course, and will be discussed in section §5.

4 Local Properties

In this section some fundamental properties arising from interactions are discussed.

Please note that everything in this section happens at only a single point in embedded space. When I am writing about particles moving, here only field points moving in embedding space are meant.

4.1 Effective Rotation Interaction

While the effect of rotation interaction depends on both, the source and destination particles' current state, i.e. their orientations, effective rotation interaction integrates out all these states and thereby corresponds to classical electric charge.

Please note that this calculation is true for the source as well as destination particles, and thus a dummy rotation vector is used instead of considering an intermediate field. Furthermore some simplifications are applied to be able to give an exact solution of the integrals.

In the following expression $\mathfrak{S}(\theta,\phi)$ acts firstly as the direction to the other particle in embedding space as well as secondly as the dummy rotation vector. In consideration of the integration (use of $\mathfrak{S}(\theta,\phi)$) and normalization due to simplification, the interaction itself is the same as in function 20 on the preceding page.

$$charge(p) = \underbrace{\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} d\phi d\theta \sin\theta}_{1} \underbrace{\sum_{A}^{\mathbb{R}(p)}\sum_{B}^{\mathbb{R}(p)}}_{2a} \underbrace{\sum_{a}^{\mathbb{O}(p)}\sum_{b}^{\mathbb{O}(p)}}_{2b} \underbrace{\begin{cases} \frac{1}{2} & \mathcal{N}(p)=1 \\ \frac{1}{5} & \mathcal{N}(p)=2 \end{cases} \begin{cases} \frac{1}{2} & A=B \\ \frac{1}{3} & A\neq B \end{cases}}_{3} \underbrace{\mathfrak{S}(\theta,\phi)\cdot\frac{P(p,B,b)-P(p,A,a)}{2}}_{4} \underbrace{\mathfrak{S}(\theta,\phi)\cdot R(p,A,a)}_{5}$$

Return value: A positive or negative scalar. To do calculations with this value it has to be multiplied by a unit vector pointing from the source to the destination particle in embedding space.

1. Integrate over a unit sphere since the other particle may be located in any direction.
2. To be able to ignore the distance to the destination particle a pairwise summation for the rotation points is applied over:

(a) all rotating vertices and

(b) all their orientation states.

3. Normalize the sums: If there is one rotating vertex it is used in combination with 2, and in the case of two rotating vertices also with another 3 orientation states ($3 + 2 = 5$). Within one vertex's orientation states ($A = B$) each position is combined with 2 orientation states otherwise with 3.

4. In the limit of large distances from the pairwise combined sources $P(p, A, a)$ and $P(p, B, b)$ to some destination point $D = r\,\mathfrak{S}(\theta, \phi)$ the difference of the norms $(\mathfrak{N}(D - P(p, A, a)) - \mathfrak{N}(D - P(p, B, b)))$ can be replaced that way.

5. Effect of $R(p, A, a)$ in direction $\mathfrak{S}(\theta, \phi)$.

Results

Calculation results show that it has to be distinguished between particles having one resp. two rotating vertices (i.e. $\mathcal{N}(p) = 1$ or $\mathcal{N}(p) = 2$).

$\mathcal{N}(p) = 1$**:** Such particles have an effective rotation charge of $-\frac{1}{8}$, 0 or $\frac{1}{8}$. The charges of $\pm\frac{1}{8}$ show to be one half of one elementary charge. It turns out that the electron is made up of two such $-\frac{1}{8}$ particles; the same is analogously true for the positron.

$\mathcal{N}(p) = 2$**:** Since rotation charge depends on the distance between rotation points $\mathcal{P}$ the overall size of the graph's play a role (i.e. $\mathcal{L}(p)$). The dependence is linear in regard of $\mathcal{L}(p)$.

$\mathcal{L}(p) = 1$**:** These particles have charges of $-\frac{1}{12}$, 0 or $\frac{1}{12}$. Quarks with one third of one elementary charge belong to these.

$\mathcal{L}(p) = 2$**:** Obviously particles of this group have charges of $-\frac{1}{6}$, 0 or $\frac{1}{6}$, and quarks with two third of one elementary charge belong to these.

$\mathcal{L}(p) = 3$**:** Here charges of $-\frac{1}{4}$, 0 and $\frac{1}{4}$ can be found. These particles have besides 0 plus or minus one elementary charge. They include the muon and tauon, which are not composite particles like electron and positron.

4.2 Integration of States

From here on again and again the interaction function $i()$ has to be integrated over Euler angles for every considered particle. This is put into some function, which are simple to use and fast as soon as these time consuming symbolic calculations are done once.

It turns out that the interaction functions' result decompose into two parts after integration. These have to be mapped to two different function types:

1. Interaction effects as absolute (A) vectors in space.

2. Effects as vectors relative (R) to the particle's triangle.

The functions bear the names *iAfs*() and *iRfs*() - "*fs*" for "functions" since *iAfs*() and *iRfs*() return functions in dependence of specifiable parameters.

If interaction function results are being used for e.g. mass calculation, the results from these functions are simply added and can be further used. But if particle's translation in embedding space is demanded, a correction is needed - see equation (15). This is taken into account in function *iATfs*() (T for translation). There's no need to define a function like *iRTfs*() since interaction vectors relative to the triangle after integration are $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$ and thus never lead to translation.

All three functions have the same parameters:

$$iAfs(ps, Xs, inf, iofNOTx) \tag{21}$$

$$iATfs(ps, Xs, inf, iofNOTx) \tag{22}$$

$$iRfs(ps, Xs, inf, iofNOTx) \tag{23}$$

ps List of particle numbers to use in integration.

Xs List of coordinates for every particle in *ps*.

inf This value is either *True* or *False* and specifies whether particles having different coordinates *Xs* are assumed to be located in infinite distance (in direction of the given coordinates) or at the given points.

iofNOTx If this parameter is *True* orientation interaction is taken into account in calculation, if not, only rotation interaction is used.

The result of these functions is a list if interaction vectors something like calculated by $i()$, but the effective ones, after integration over Euler angles.

For details please refer to the corresponding Mathematica files.

4.3 Locally Bound States

> Unfortunately I was not yet able to find and develop a comprehensive algorithm to automatically calculate all physical states that occur in nature and are known from SM. According to this theory many of these appear in the course of time and thus are not immediately obvious - other, not known states even still could emerge while being probably very rare. Therefore in examining bound states (clumping of particle fields to e.g. hadrons) and calculating masses, it's in part necessary to make educated guesses.

Orientation and rotation interactions are added up for every particle to form a single 3-vector as in function $i()$ (17). This vector describes the particles' transformation, which is applied by a rotation matrix and a translation like in function $i\Delta()$ (15). From $i\Delta()$ one can read off that due to the spaces' sizes transformations are running in, translations have a factor h_f, rotations of attached vertices have a factor h_a and rotations of the triangles have a factor h_t. Considering the magnitude of these constants one immediately sees that translation of the entire graph is negligible compared to rotation of attached vertices, and rotation of attached vertices is negligible in comparison to rotation of the triangle. That means, one can integrate the translation effect of interactions over all possible rotation states of the triangle and attached vertices before one has to consider what translation means. Thus it's sufficient, to only use function $iATfs()$ here.

Still $iATfs()$ needs the coordinates for every particle as a parameter, but here it's only of interest how particles interact when they are located in a certain distance. There's also some ambiguity in choosing coordinates of more than two particles. That's why hereafter a measure for binding is introduced. It specifies how fast the particles would move towards in dependence of two parameters: First the constituents of a composite particle in distance r to each other and second composite particles as a whole to each other in distance R. Of course, this is not a physical quantity, but a useful measure for this kind of binding. Function $iB()$ does that job:

$$iB(pdgnumber, antiparticle, iofNOTx) \tag{24}$$

$$iB(ps, antiparticle, iofNOTx) \tag{25}$$

pdgnumber Particle Data Group's Monte Carlo particle number, or

ps a list of lists - for every composite particle a list of the constituent's particle numbers.

antiparticle Take antiparticle (second entry in the list of lists of particle numbers) into account.

iofNOTx If this parameter is *True* orientation interaction is taken into account in calculation, if not, only rotation interaction is used.

These functions return a list with one entry for every fundamental particle. Every entry is another list, with two entries, containing as first element the binding within the composite particle and as second element the binding to the other composite particle.

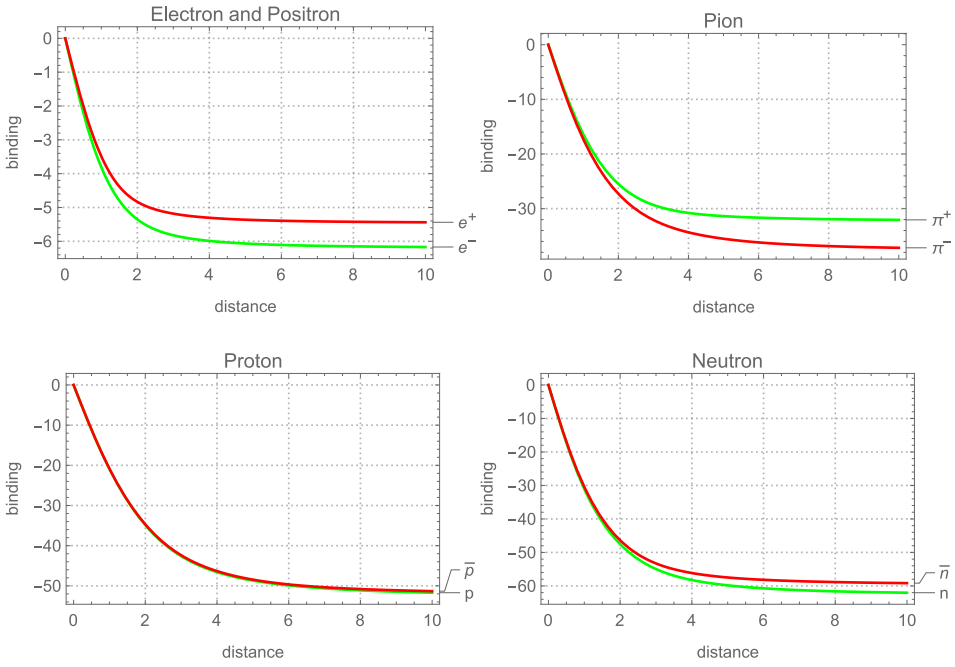
$$iBG(pdgnumber, antiparticle, iofNOTx) \tag{26}$$

Function $iBG()$ does the same as above but sums the translations of individual particles for every composite particle.

4.3.1 Isolated Bound Particles are Stable

Particles' translation in embedding space is composed of contributions by orientation and rotation interaction. Orientation interaction's effect is exclusively attractive, and rotation interaction's effect can be attractive as well as repulsive. Since it turns out that orientation interaction is stronger than rotation interaction, isolated particles in embedding space always attract each other. And that means isolated composite particles are all the time stable.

A few examples using $iBG(pdgnumber, False, True)$:



All binding values are always negative and with it interaction is attractive.

This behavior is only true as long as there are no other effects acting, like interactions with approaching other particles and translations as a consequence of wave propagation.

4.3.2 Long-Lasting Particles are More Stable than Antiparticles

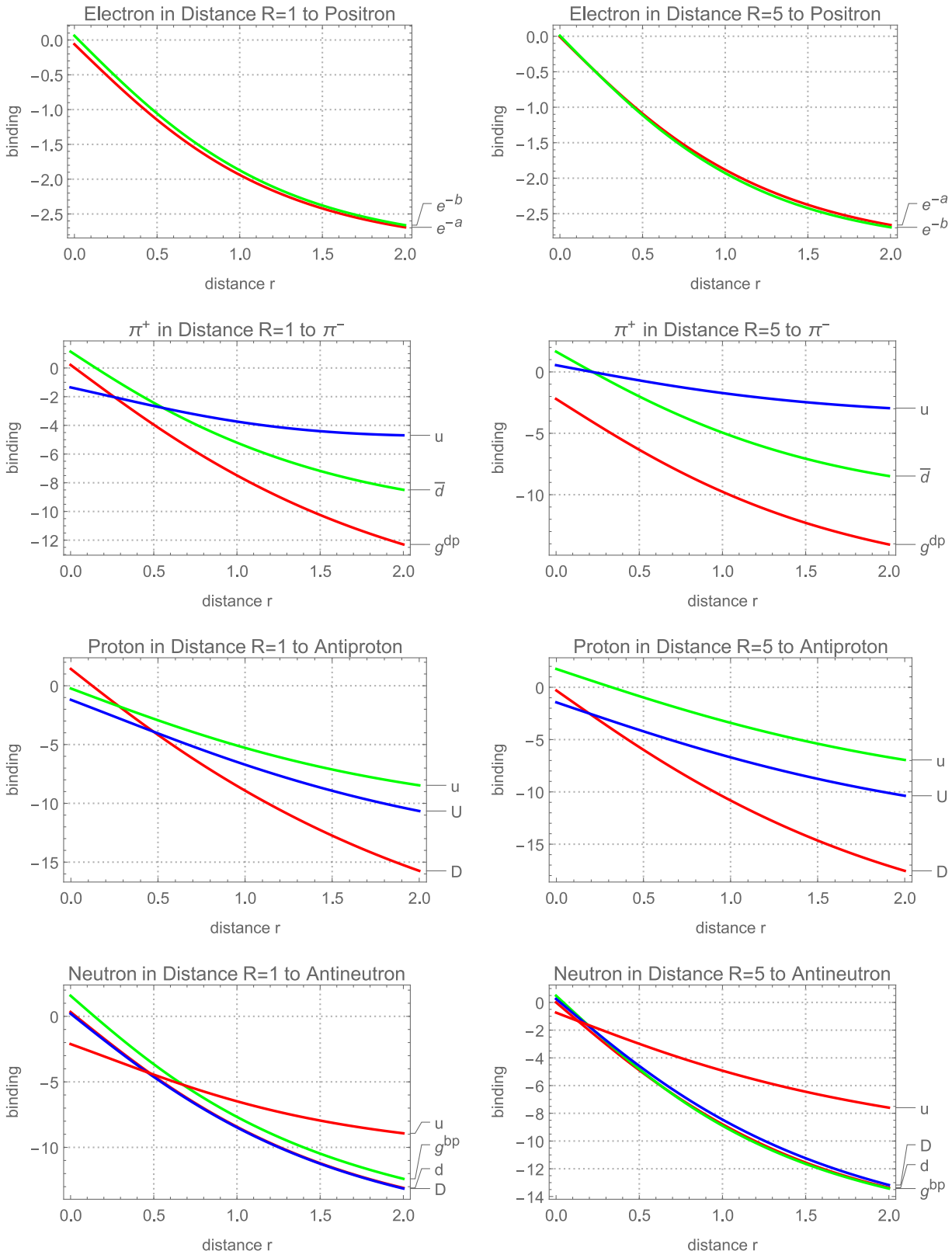
Examining the plots in section 4.3.1 makes a presumption appear: Long-lasting particles are stronger bound than their antiparticles, and thus should be more stable. (This is not true for all hadrons in respect of their classification as particles or antiparticles.) But again, here not stability with respect to decay is meant, but rather stability of the composite field itself, it's existence, not it's excitation.

In consequence to this in the early universe particle field's excitations could have got a better chance to build out and/or survive than antiparticle field's.

4.3.3 Distance of Particles Affects Bound States

While isolated composite particles are always stable, they may be not when other particles are taken into account and/or they come close in embedding space.

For the same particles as in section 4.3.1 function $iB(pdgnumber, True, True)$ gives the following results, if plotted for $R = \{1, 5\}$ and $r = 0 \ldots 2$:

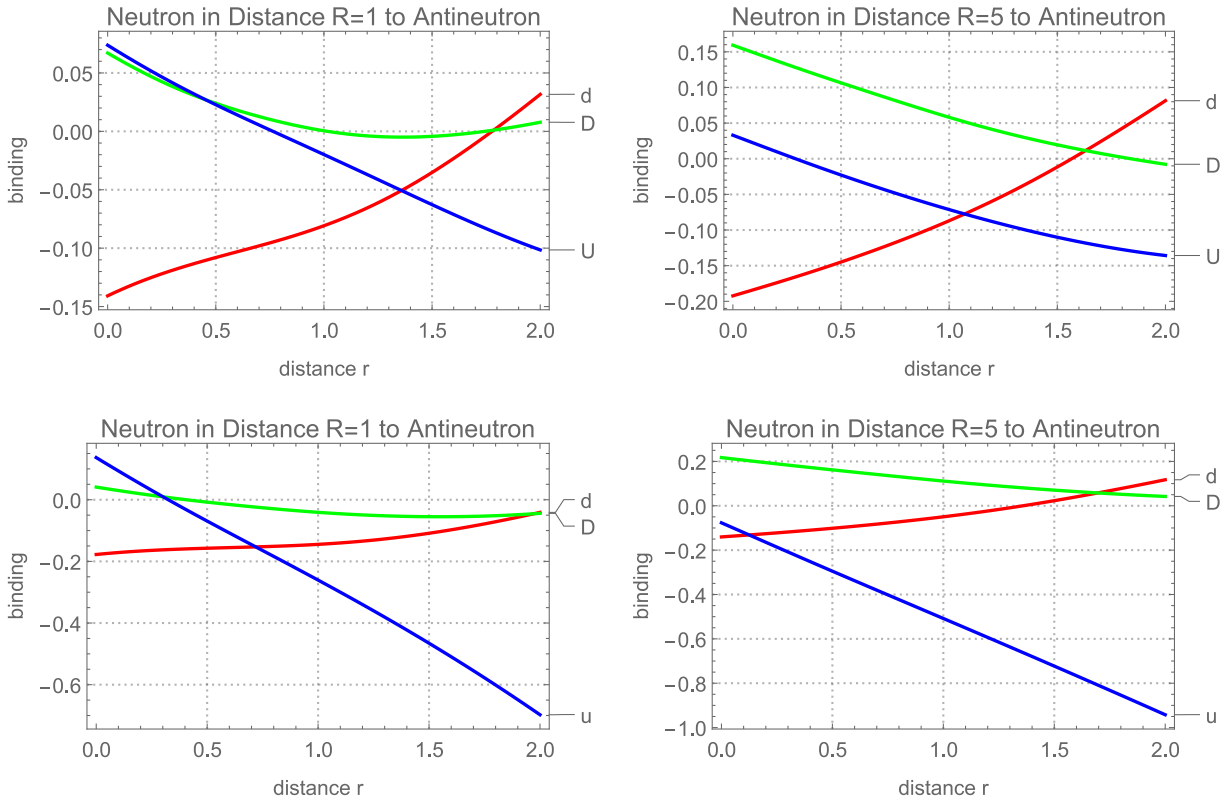


As one can read off the plots, in some distances antiparticles make particles unstable in embedding space. This appears as some constituent particles have positive values, meaning they are attracted more by the antiparticle than by the particle's constituents.

In this regard it naturally plays a role how composite particles are moving around in embedding space in dependence of other particles and due to wave induced transformations. But this shows that some kinds of composite particles are more stable than others in a complicated environment, and bound states can change members and build other composite particles. This should play a role in early universe's field formation as well.

4.3.4 Examples for Unstable Bound Particles

For instance for neutron there are more candidates than only one in regard of composition. Here, two examples with the correct quark content but without a particle g^{type} are given. These show a fairly unstable composition of these other kinds of "neutron" fields.



4.4 Local Mass Contributions

As stated in section 3.1, calculation of particles' mass requires several parts. In this section, only local contributions are considered, and thus with the function *massL*() described here, only the mass of non-composite particles can be calculated accurately. Muon and tauon are such particles; they are the simplest when comparison to SM is demanded. Also for electron and positron useful values can be found, while still a contribution is missing.

Only parameter of the function is *ps*. This is a list of lists of particle numbers, e.g. $\{\{57\},\{a(57)\}\}$ for muon and it's antiparticle, or $\{\{5,13\},\{8,18\}\}$ for electron and positron.

The most important part of the function given here, are functions *iAfs*() and *iRfs*(). For these, please refer to section 4.2. These functions are called with parameters

1. *ps* (flattened, here to e.g. $\{57, a(57)\}$),
2. x being the coordinates for every sub-list of particles identical but different among each other (also flattened to e.g. $\{\begin{pmatrix}0&0&0\end{pmatrix}^T, \begin{pmatrix}0&0&1\end{pmatrix}^T\}$),
3. *True* to treat different coordinates as being infinitely far from each other[1] and
4. *False* to exclude orientation interaction between particles having not the same position.

That means, *massL* positions all particles in *ps* at x and calculates $i()$ while integrating all orientations of all particles in regard of Euler angles, and sums the parts.

Symbol n is the number of lists in *ps* (number of particle groups with different positions) and c is the total number of particles.

[1]This is an approximation. It is not known, how far particles are away from each other in embedding space, because that's a dynamic quantity. But a sufficiently exact known measured value for masses would allow to calculate in which configuration particles reside in embedding space.

$$massL(ps) = \frac{1}{n}\sum_{p}^{c}\left(\mathfrak{N}(iAfs_p) + \mathfrak{N}(iRfs_p)\right) \tag{27}$$

While all other particles would contribute as well, their effect vanishes due to mean symmetry over all Euler angles and consideration of particles and antiparticles.

$massL()$ performs exact, symbolic calculations, but the returned expressions are extensive. Therefore the numerical values are given. Muon: 21828.81453208565 Tauon: 367113.2875047015.

5 Global Properties

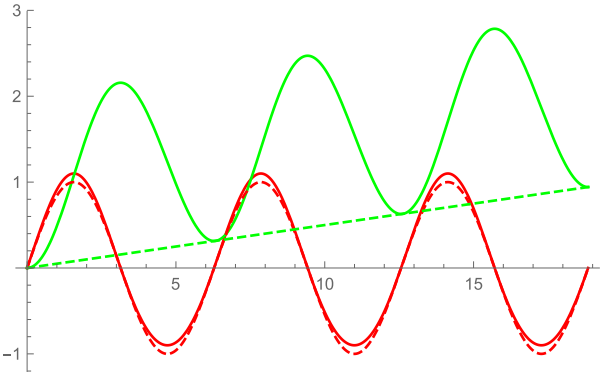
[in preparation]

5.1 Confinement

I could not yet calculate confinement but I can explain an effect that obviously leads to wave properties that confine particles.

In this theory electric charge is not constant in time; it varies with the rotation of particles simply put sine-shaped with a little tendency to positive or negative charge, something like $(\Delta + q\sin(t))\sin(t)/\Delta$ for large Δ.

The following plot shows the principle (x-axis is time t; y-axis is charge resp. the charge's effect): The red dashed line is just a sine-function to compare with, while the red solid line shows the principle of charge in this theory. It represents a positive charge, thus there's a little positive tendency. Integrating this for t from 0 to some value gives the effect of the charge, an ascending sine-function, shown as a green solid line. When ignoring very short periods of time the sine-function oscillates in, one has the overall effect shown as the green dashed line.



This is what happens at a single point. But the same is true for "force carrying" fields reacting to the charge while propagating away with the speed of light. So, there is some periodicity observable in these fields at a wave length of $\lambda_{force\,field} = c/\left(E_{source}/h\right)$.

However, this correlation is only true for charged source fields with $\lambda_{source} \gg c/\nu_{source}$ and thus showing negligible rotation along the way of propagation. When $\lambda_{source} \ll c/\nu_{source}$ this rotation is not negligible and thus that effect breaks down, because the shortness of the source field's wave length exposes the force field quickly to all orientations of the source particle.

This structure of the force field is transferred to affected matter waves. Long wave lengths defining the particle's large-scale position are restricted to wavelengths of $\lambda_{force\,field}$, but short wave lengths are not restricted like this and let wave packets propagate within $\lambda_{force\,field}$. Hence particles are confined to a blurred radius of something at $c\,h/\left(2E\right)$. For proton's mass from local contributions: $r = 6.56588 \times 10^{-16} m$.

5.2 Globally Bound States

[in preparation]

5.3 Global Mass Contributions

This section builds up on local mass contributions as discussed in section 4.4 by adding up interactions mediated over fields.

[in preparation]

Part III
Verification of the Theory

6 Electric Charges

As per section 4.1 this theory predicts particles with electric charge of 0, $\pm\frac{1}{8}$ and $\pm n\frac{1}{12}; n \in \mathbb{N}$.

It turns out, that $\pm e$ (elementary charge) equals $\pm\frac{1}{4}$ in this theory. Particles with charge $\pm\frac{1}{8}$ are bound together in pairs and form electron and positron with a composite charge of each $\pm\frac{1}{4}$. That equals the properties of the SM.

Until now I did not find a binding reason for particles with charges $\pm n\frac{1}{12}; n > 3 \in \mathbb{N}$ should not exist, but these particles' masses would increase strongly. Nevertheless this has to be studied further.

7 Particle Masses

> Since on the one hand it was not yet possible to make a good and usable calculation for mediated interactions, and on the other hand development of an algorithm to determine existing bound states (including mixing and superposition), e.g. for hadrons, is not yet done, in some aspects of the following particles, educated guesses in regard of particles' composition have to be made.

In this section m is used for masses from SM in eV or MeV, and M is used for the corresponding local part ($massL()$) in this theory. Because of still missing non-local mass contributions a very coarse classical estimate for these are shown here as well - simply the Coulomb energy ($coulombEnergy()$) integrated over a ball with radius of composite particles' assessed radius ($particleRadius()$) as ΔM. Please refer to the Mathematica notebooks for these functions.

7.1 Leptons

The formula for calculation of muon and tauon mass is already fully elaborated. (Please note that muon mass is used for calculation of $SIeV$, the constant to convert a quantity from this theory to eV. Therefore $M/m = 1$ and $M - m = 0$ is compelling.)

	M	M (MeV)	m (MeV)	M/m	M-m (MeV)
$\mu^{\pm}$	21828.81453208565	105.6583755	105.6583755	1.	0.
$\tau^{\pm}$	367113.2875047015	1776.944576	1776.86	1.00005	0.08

For electron and positron a contribution from the binding of the constituent particles is missing.

	M	M (eV)	m (eV)	M/m	M-m (eV)
$e^{\pm}$	105.9006594570622	512592.7304098458	510998.95000	1.0031189505	1593.78041
	$\Delta M = -1785.99eV$	510806.7396417846		0.99962385371	-192.21036

7.2 Mesons

Only a few mesons are looked at here.

When examining the particles of this theory, there are in total 18 uncharged ones with two rotating vertices, and thus having some mass. These are named g^{type}, while *type* stands for the particle each is most similar to in regard of their structure. When restricting to those of them having $\mathcal{L} = 1$, thus the ones with the least mass, following particles remain (in order of mass):

no.	name	mass	similar to
7	g^{dn}	25343.4979	down quark, negative charge
4	g^{dp}	25816.6664	down quark, positive charge
10	g^{sn}	96160.2092	strange quark, negative charge
1	g^{sp}	98303.9009	strange quark, positive charge
17	g^{bn}	155814.0534	bottom quark, negative charge
16	g^{bp}	158515.7247	bottom quark, positive charge

Extending these lightest g-particles with the lightest quarks, one gets composite particles as follows:

PDG	name	quarks	M	M	m	M/m	M-m	ΔM
				(MeV)	(MeV)		(MeV)	(MeV)
111	π^0	$\{\{g^{dp}, d, \overline{d}\}, \{g^{dn}, \overline{u}, u\}\}$	29571.521	143.1355	134.9766	1.060447	8.1589	*
211	π^+	$\{g^{dp}, \overline{d}, u\}$	29573.996	143.14750	139.57018	1.0256310	3.57732	-0.50
-211	π^-	$\{g^{dn}, d, \overline{u}\}$	29573.996	143.14750	139.57018	1.0256310	3.57732	-0.50
321	K^+	$\{g^{sp}, \overline{s}, u\}$	103801.022	502.430	493.677	1.01773	8.753	-1.75
-321	K^-	$\{g^{sn}, s, \overline{u}\}$	103801.022	502.430	493.677	1.01773	8.753	-1.75
311	K^0	$\{g^{sp}, \overline{s}, d\}$	101084.172	489.279	497.648	0.983184	-8.369	*
-311	$\overline{K}^0$	$\{g^{sn}, s, \overline{d}\}$	101084.172	489.279	497.648	0.983184	-8.369	*
113	ρ^0	$\{\{g^{bp}, d, \overline{d}\}, \{g^{bn}, \overline{u}, u\}\}$	161151.709	780.0	775.5	1.006	4.5	*
213	ρ^+	$\{g^{bp}, \overline{d}, u\}$	161158.815	780.1	775.5	1.006	4.6	-2.72
-213	ρ^-	$\{g^{bn}, d, \overline{u}\}$	161158.815	780.1	775.5	1.006	4.6	-2.72

*) These values can't be calculated for uncharged particles.

7.3 Baryons

For baryons in principle the same is done as for mesons, except for proton and it's antiparticle.

Nucleons

PDG	name	quarks	M	M	m	M/m	M-m	ΔM
				(MeV)	(MeV)		(MeV)	(MeV)
2212	p	$\{D, u, U\}$	195060.56790	944.15492411	938.27208816	1.0062698614	5.88283595	-3.29
-2212	$\overline{p}$	$\{\overline{D}, \overline{u}, \overline{U}\}$	195060.56790	944.15492411	938.27208816	1.0062698614	5.88283595	-3.29
2112	n	$\{g^{bp}, d, D, u\}$	194088.5573	939.4500851	939.5654205	0.9998772460	-0.1153354	
-2112	$\overline{n}$	$\{g^{bn}, \overline{d}, \overline{D}, \overline{u}\}$	194088.5573	939.4500851	939.5654205	0.9998772460	-0.1153354	

Lambda

PDG	name	quarks	M	M	m	M/m	M-m	ΔM
				(MeV)	(MeV)		(MeV)	(MeV)
3122	Λ	$\{g^{up}, s, d, U\}$	235387.26	1139.349	1115.683	1.021212	23.666	-3.97
-3122	$\overline{\Lambda}$	$\{g^{un}, \overline{s}, \overline{d}, \overline{U}\}$	235387.26	1139.349	1115.683	1.021212	23.666	-3.97

Sigma

PDG	name	quarks	M	M	m	M/m	M-m	ΔM
				(MeV)	(MeV)		(MeV)	(MeV)
3222	Σ^+	$\{g^{up}, s, u, U\}$	238097	1152.46	1189.37	0.968969	-36.91	-4.02
-3222	$\overline{\Sigma}^-$	$\{g^{un}, \overline{s}, \overline{u}, \overline{U}\}$	238097	1152.46	1189.37	0.968969	-36.91	-4.02
3212	Σ^0	$\{g^{up}, S, d, u\}$	257578.5	1246.762	1192.642	1.045378	54.12	
-3212	$\overline{\Sigma}^0$	$\{g^{un}, \overline{S}, \overline{d}, \overline{u}\}$	257578.5	1246.762	1192.642	1.045378	54.12	
3112	Σ^-	$\{g^{dp}, S, d, D\}$	239976.7	1161.563	1197.449	0.9700314	-35.89	-4.05
-3112	$\overline{\Sigma}^+$	$\{g^{dn}, \overline{S}, \overline{d}, \overline{D}\}$	239976.7	1161.563	1197.449	0.9700314	-35.89	-4.05

Delta

PDG	name	quarks	M	M	m	M/m	M-m	ΔM
				(MeV)	(MeV)		(MeV)	(MeV)
2114	$\Delta(1232)^0 P_{33}$	$\{g^{up}, d, D, U\}$	2651×10^2	1283	1232	1.042	51	-4.47
-2114	$\overline{\Delta}(1232)^0 P_{33}$	$\{g^{un}, \overline{d}, \overline{D}, \overline{U}\}$	2651×10^2	1283	1232	1.042	51	-4.47

Xi

PDG	name	quarks	M	M (MeV)	m (MeV)	M/m	M-m (MeV)	ΔM (MeV)
3322	Ξ^0	$\{g^{sp}, s, S, u\}$	284618	1377.64	1314.83	1.04777	62.81	
-3322	$\overline{\Xi}^0$	$\{g^{sn}, \overline{s}, \overline{S}, \overline{u}\}$	284618	1377.64	1314.83	1.04777	62.81	
3312	Ξ^-	$\{g^{sn}, s, S, d\}$	281913	1364.55	1321.31	1.03272	43.24	-4.75
-3312	$\overline{\Xi}^+$	$\{g^{sp}, \overline{s}, \overline{S}, \overline{d}\}$	281913	1364.55	1321.31	1.03272	43.24	-4.75

Theta

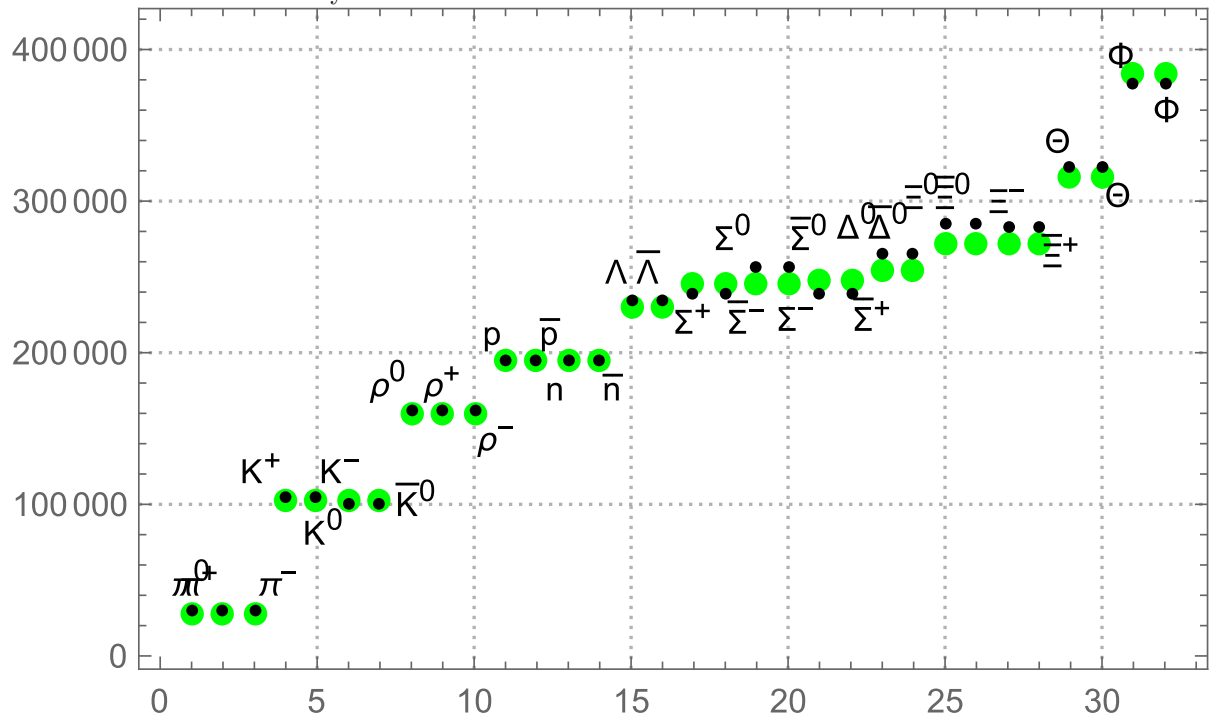
PDG	name	quarks	M	M (MeV)	m (MeV)	M/m	M-m (MeV)	ΔM (MeV)
9221132	$\Theta(1540)$	$\{g^{sp}, g^{dp}, \overline{s}, d, D, u, U\}$	32175×10^1	1557.4	1533.6	1.0155	23.8	-5.43
-9221132	$\overline{\Theta}(1540)$	$\{g^{sn}, g^{dn}, s, \overline{d}, \overline{D}, \overline{u}, \overline{U}\}$	32175×10^1	1557.4	1533.6	1.0155	23.8	-5.43

Phi

PDG	name	quarks	M	M (MeV)	m (MeV)	M/m	M-m (MeV)	ΔM (MeV)
9331122	$\Phi(1860)$	$\{g^{bn}, s, S, d, D, \overline{u}\}$	3781×10^2	1830	1862	0.9829	-32	-6.38
-9331122	$\overline{\Phi}(1860)$	$\{g^{bp}, \overline{s}, \overline{S}, \overline{d}, \overline{D}, u\}$	3781×10^2	1830	1862	0.9829	-32	-6.38

7.4 Hadrons Plotted

The following plot shows all above listed hadrons with their masses. Green points are measured values from SM, the black points are the values from *massL*(), so, only the local mass contributions are considered. The units are the ones of this theory.



Part IV
Appendix

8 SI Units

In this section conversion factors from this theory's quantities to the SI-System are given. They are named as SIx where x is the SI-unit and applied like: $value_{NAEG-ToE} * SIx = value_{SI}$.

8.1 Time and Length

Despite it seems to make sense to associate time and length units with Planck time and Planck length this assumption should be reviewed as soon as more calculations are done.

$$SIs = 5.391247 \times 10^{-44}$$

$$SIm = 1.616255 \times 10^{-35}$$

8.2 Energy and Mass

In the standard model muon mass is very accurately known and at the same time it can be calculated relatively simple in this theory even without the use of the constant $\ddot{u}$ just by applying the formulas to the graphs of particle $57 = \mu^-$ and it's antiparticle $a(57) = 60 = \mu^+$. This makes it an ideal starting point for energy and mass conversion factors.

Muon mass calculated by this theory is $M_\mu = 21828.81453208565$.

Using muon mass of the standard model in eV gives:

$$SIeV = 105658375.5\, eV/M_\mu = 4840.316699$$

Using the conversion from eV to J gives:

$$SIJ = 1602176634 \times 10^{-28} \frac{J}{eV} SIeV = 7.755042316 \times 10^{-16}$$

And additionally using the speed of light gives:

$$SIkg = SIeV \frac{1602176634 \times 10^{-28} \frac{J}{eV}}{c^2 \frac{m^2}{s^2}} = 8.628648268 \times 10^{-33}$$

9 Value of $\ddot{u}$

Muon mass is well measured in eV and there's a straight forward calculation corresponding to that value in this theory M_μ. In combination with Planck's constant and its analogous h_t (see section §10) as well as *SIs* it's possible to calculate $\ddot{u}$.

Frequency of muon due to it's mass in s^{-1} is calculated by $f_\mu = m_\mu \frac{c^2}{h}$ (m_μ = muon mass, c = speed of light, h = Planck's constant). The number of Planck times, which is equal to time units in this theory, for 2π thus is $n_\mu = \frac{1}{f_\mu}/SIs$. Taking this number multiplied by M_μ must be equal to the size of each particle's triangle's degrees of freedom, therefore $\frac{1}{h_t} = n_\mu M_\mu$. Solving this gives $\ddot{u} = 1.26706 \times 10^{75}$.

10 Analogous to Planck Constant

Every particle's triangle and attached vertices have degrees of rotational freedom whose size is (by definition as reciprocal) $1/h_t$ resp. $1/h_a$. In addition the remaining degrees of freedom have size $1/h_f$. The latter ones are dimensions that are independent of each other thus their sizes are also independent of each other:

$$h_f = 1/\ddot{u}$$

Former ones are dependent of others, thus the volume of these spaces are in total $\ddot{u}$. For the triangle the rotation space is of size $4\pi * 2\pi$ (spherical orientation; three rotations):

$$\ddot{u} = 4\pi \left(\frac{1}{h_t}/(2\pi)\right)^2 2\pi \left(\frac{1}{h_t}/(2\pi)\right)$$

$$h_t = \frac{1}{\pi^{1/3} \ddot{u}^{1/3}}$$

For attached vertices the rotation space is of size 4π (spherical orientation; two rotations):

$$\ddot{u} = 4\pi \left(\frac{1}{h_a}/(2\pi)\right)^2$$

$$h_a = \frac{1}{\sqrt{\pi}\sqrt{\ddot{u}}}$$

All interaction induced transformations in these spaces have to be multiplied by h_t, h_a resp. h_f.

11 Particle List

Particle names are chosen in the style of standard model, but they don't necessarily meet exact equality. Note that there are upper and lower case variants used.

no.	anti-no.	name	$\mathcal{L}$ (length)	$\mathbb{E}$ (edges/graph structure)	charge	winding	mass
1	10	g^{sp}	1	$1\to2, 2\to3, 3\to1, 4\to1, 5\to1$	0	2	98303.9009
2	11	s	1	$1\to2, 2\to3, 3\to1, 4\to2, 5\to2$	$-\frac{1}{12}$	1	3236.4419
3	12	$\bar{S}$	1	$1\to2, 2\to3, 3\to1, 4\to3, 5\to3$	$\frac{1}{12}$	2	182243.1804
4	7	g^{dp}	1	$1\to2, 2\to3, 3\to4, 4\to2, 5\to2$	0	2	25816.6664
5	8	e^{-a}	1	$1\to2, 2\to3, 3\to4, 4\to2, 5\to3$	$-\frac{1}{8}$	1	56.1450
6	9	f^{+a}	1	$1\to2, 2\to3, 3\to4, 4\to2, 5\to4$	$\frac{1}{8}$	1	55.8975
7	4	g^{dn}	1	$1\to2, 2\to4, 3\to2, 4\to3, 5\to2$	0	2	25343.4979
8	5	e^{+a}	1	$1\to2, 2\to4, 3\to2, 4\to3, 5\to3$	$\frac{1}{8}$	1	56.1450
9	6	f^{-a}	1	$1\to2, 2\to4, 3\to2, 4\to3, 5\to4$	$-\frac{1}{8}$	1	55.8975
10	1	g^{sn}	1	$1\to3, 2\to1, 3\to2, 4\to1, 5\to1$	0	2	96160.2092
11	2	$\bar{s}$	1	$1\to3, 2\to1, 3\to2, 4\to2, 5\to2$	$\frac{1}{12}$	1	3188.1830
12	3	S	1	$1\to3, 2\to1, 3\to2, 4\to3, 5\to3$	$-\frac{1}{12}$	2	179412.2625
13	18	e^{-b}	1	$1\to3, 2\to3, 3\to4, 4\to2, 5\to2$	$-\frac{1}{8}$	1	51.9068
14	19	d	1	$1\to3, 2\to3, 3\to4, 4\to2, 5\to3$	$-\frac{1}{12}$	1	641.1867
15	20	γ^{an}	1	$1\to3, 2\to3, 3\to4, 4\to2, 5\to4$	0	1	56.3272
16	17	g^{bp}	1	$1\to3, 2\to3, 3\to4, 4\to5, 5\to3$	0	2	158515.7247
17	16	g^{bn}	1	$1\to3, 2\to3, 3\to5, 4\to3, 5\to4$	0	2	155814.0534
18	13	e^{+b}	1	$1\to3, 2\to4, 3\to2, 4\to3, 5\to2$	$\frac{1}{8}$	1	51.9068
19	14	$\bar{d}$	1	$1\to3, 2\to4, 3\to2, 4\to3, 5\to3$	$\frac{1}{12}$	1	632.9925
20	15	γ^{ap}	1	$1\to3, 2\to4, 3\to2, 4\to3, 5\to4$	0	1	56.3272
21	24	f^{+b}	1	$1\to4, 2\to3, 3\to4, 4\to2, 5\to2$	$\frac{1}{8}$	1	49.1223
22	25	γ^{bn}	1	$1\to4, 2\to3, 3\to4, 4\to2, 5\to3$	0	1	51.5842
23	26	$\bar{D}$	1	$1\to4, 2\to3, 3\to4, 4\to2, 5\to4$	$\frac{1}{12}$	2	33177.0271
24	21	f^{-b}	1	$1\to4, 2\to4, 3\to2, 4\to3, 5\to2$	$-\frac{1}{8}$	1	49.1223
25	22	γ^{bp}	1	$1\to4, 2\to4, 3\to2, 4\to3, 5\to3$	0	1	51.5842
26	23	D	1	$1\to4, 2\to4, 3\to2, 4\to3, 5\to4$	$-\frac{1}{12}$	2	32703.8581
27	28	b	1	$1\to4, 2\to4, 3\to4, 4\to5, 5\to3$	$-\frac{1}{12}$	1	3236.4419
28	27	$\bar{b}$	1	$1\to4, 2\to4, 3\to5, 4\to3, 5\to4$	$\frac{1}{12}$	1	3188.1830
29	30	$\bar{B}$	1	$1\to5, 2\to5, 3\to4, 4\to5, 5\to3$	$\frac{1}{12}$	2	156380.9781
30	29	B	1	$1\to5, 2\to5, 3\to5, 4\to3, 5\to4$	$-\frac{1}{12}$	2	154190.4945
31	36	g^{cp}	2	$1\to2, 2\to3, 3\to1, 4\to1, 5\to1$	0	2	327407.7133
32	37	$\bar{c}$	2	$1\to2, 2\to3, 3\to1, 4\to2, 5\to2$	$-\frac{1}{6}$	1	27955.2043
33	38	C	2	$1\to2, 2\to3, 3\to1, 4\to3, 5\to3$	$\frac{1}{6}$	2	1527758.8431
34	35	g^{up}	2	$1\to2, 2\to3, 3\to4, 4\to2, 5\to2$	0	2	73714.0087
35	34	g^{un}	2	$1\to2, 2\to4, 3\to2, 4\to3, 5\to2$	0	2	71821.3397
36	31	g^{cn}	2	$1\to3, 2\to1, 3\to2, 4\to1, 5\to1$	0	2	318832.9588
37	32	c	2	$1\to3, 2\to1, 3\to2, 4\to2, 5\to2$	$\frac{1}{6}$	1	27762.1254
38	33	$\bar{C}$	2	$1\to3, 2\to1, 3\to2, 4\to3, 5\to3$	$-\frac{1}{6}$	2	1516435.1592
39	42	$\bar{u}$	2	$1\to3, 2\to3, 3\to4, 4\to2, 5\to3$	$-\frac{1}{6}$	1	3358.3998
40	41	g^{tp}	2	$1\to3, 2\to3, 3\to4, 4\to5, 5\to3$	0	2	2579072.3674
41	40	g^{tn}	2	$1\to3, 2\to3, 3\to5, 4\to3, 5\to4$	0	2	2566154.9019
42	39	u	2	$1\to3, 2\to4, 3\to2, 4\to3, 5\to3$	$\frac{1}{6}$	1	3325.6197
43	44	U	2	$1\to4, 2\to3, 3\to4, 4\to2, 5\to4$	$\frac{1}{6}$	2	159702.5122
44	43	$\bar{U}$	2	$1\to4, 2\to4, 3\to2, 4\to3, 5\to4$	$-\frac{1}{6}$	2	157809.8354
45	46	$\bar{t}$	2	$1\to4, 2\to4, 3\to4, 4\to5, 5\to3$	$-\frac{1}{6}$	1	64146.4922
46	45	t	2	$1\to4, 2\to4, 3\to5, 4\to3, 5\to4$	$\frac{1}{6}$	1	63903.4030
47	48	T	2	$1\to5, 2\to5, 3\to4, 4\to5, 5\to3$	$\frac{1}{6}$	2	2740084.4602
48	47	$\bar{T}$	2	$1\to5, 2\to5, 3\to5, 4\to3, 5\to4$	$-\frac{1}{6}$	2	2728840.3206

no.	anti-no.	name	$\mathcal{L}$ (length)	$\mathbb{E}$ (edges/graph structure)	charge	winding	mass
49	54	$g^{\tau p}$	3	$1\to2, 2\to3, 3\to1, 4\to1, 5\to1$	0	2	1656878.7925
50	55	τ^-	3	$1\to2, 2\to3, 3\to1, 4\to2, 5\to2$	$-\frac{1}{4}$	1	367323.1684
51	56	T^+	3	$1\to2, 2\to3, 3\to1, 4\to3, 5\to3$	$\frac{1}{4}$	2	18744649.7902
52	53	$g^{\mu p}$	3	$1\to2, 2\to3, 3\to4, 4\to2, 5\to2$	0	2	252274.8406
53	52	$g^{\mu n}$	3	$1\to2, 2\to4, 3\to2, 4\to3, 5\to2$	0	2	248016.3226
54	49	$g^{\tau n}$	3	$1\to3, 2\to1, 3\to2, 4\to1, 5\to1$	0	2	1637585.5232
55	50	τ^+	3	$1\to3, 2\to1, 3\to2, 4\to2, 5\to2$	$\frac{1}{4}$	1	366888.7155
56	51	T^-	3	$1\to3, 2\to1, 3\to2, 4\to3, 5\to3$	$-\frac{1}{4}$	2	18719171.4938
57	60	μ^-	3	$1\to3, 2\to3, 3\to4, 4\to2, 5\to3$	$-\frac{1}{4}$	1	21858.3511
58	59	g^{wp}	3	$1\to3, 2\to3, 3\to4, 4\to5, 5\to3$	0	2	225885562.5528
59	58	g^{wn}	3	$1\to3, 2\to3, 3\to5, 4\to3, 5\to4$	0	2	225850888.7735
60	57	μ^+	3	$1\to3, 2\to4, 3\to2, 4\to3, 5\to3$	$\frac{1}{4}$	1	21784.5869
61	62	M^+	3	$1\to4, 2\to3, 3\to4, 4\to2, 5\to4$	$\frac{1}{4}$	2	978140.8379
62	61	M^-	3	$1\to4, 2\to4, 3\to2, 4\to3, 5\to4$	$-\frac{1}{4}$	2	973882.3112
63	64	w^-	3	$1\to4, 2\to4, 3\to4, 4\to5, 5\to3$	$-\frac{1}{4}$	1	8077862.7209
64	63	w^+	3	$1\to4, 2\to4, 3\to5, 4\to3, 5\to4$	$\frac{1}{4}$	1	8077197.3484
65	66	W^+	3	$1\to5, 2\to5, 3\to4, 4\to5, 5\to3$	$\frac{1}{4}$	2	335684986.2864
66	65	W^-	3	$1\to5, 2\to5, 3\to5, 4\to3, 5\to4$	$-\frac{1}{4}$	2	335653878.7851

12 Functions

This is a list of all used functions. For each function a short description as well as an example of the return value is given. Reference implementations for Mathematica can be found in the corresponding files.

To reduce writing some 15×15-, 3×15- and 15×3-matrices are printed as 5×5-, 1×5 resp. 5×1-matrices; each entry of these has to be multiplied by the 3×3-identity matrix. These matrices are marked by the index 15.

$\mathbb{F}_p$ Set of particle functions containing for every particle 15 functions of the form $Gpc(t,v,w,x,y,z)$, where p is the particle number and c the coordinate as one of the letters from a to o. The first parameter is time, the others are spatial coordinates $\mathbb{C}$.

Example: $\mathbb{F}_{13} = \begin{pmatrix} G13a(t,v,w,x,y,z) & G13b(\ldots) & \ldots & G13o(t,v,w,x,y,z) \end{pmatrix}^T$
See also $\mathbb{C}$

$\mathbb{G}_p$ Set of all particles containing each first the edge's length and second the graph's adjacency matrix.

Example (for particle $e^{-a}=5$): $\mathbb{G}_5 = \left\{ 1, \begin{pmatrix} 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&1&0&0&0\\ 0&0&1&0&0 \end{pmatrix} \right\}$

See also $\mathcal{L}(p)$ and $\mathcal{A}(p)$

12.1 Auxiliary Functions (mostly gothic style letters)

12.1.1 Miscellaneous

$\delta(a,b)$ Kronecker delta function.

$\mathfrak{N}(v)$ Euclidean norm of v.

$\mathfrak{p}(v,n)$ Projection of vector v on the not pre-normalized vector n, with possible sign inversion so that the result points in direction of n.

$\mathfrak{P}(v,n)$ Projection of vector v on the already normalized vector n.

12.1.2 Coordinates

$\mathbb{C}$ Coordinates of the wave functions. x, y and z correspond to the dimensions of our world of experience.

Example (there are always five coordinates): $\mathbb{C} = \{v,w,x,y,z\}$

Note: I am not yet sure if $\mathbb{C} = \{v, w, x, y, z\}$ is correct. - These five coordinates are the spatial degrees of freedom of each graph and there are $\mathcal{C}$ distinct graphs. Therefore my uncertainty is whether these $5\mathcal{C}$ degrees of freedom collapse to only 5 or not. Unfortunately I was not yet able to do calculations which allow to distinguish this. If these degrees of freedom don't collapse it applies: $\mathbb{C} = \{v_i, w_i, x_i, y_i, z_i\}\,;\, i \in \{1 \dots \mathcal{C}\}$

$\mathfrak{S}(\theta, \phi)$ Spherical coordinates of unit radius.

Example: $\mathfrak{S}(\pi/2, \pi/3) = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}^T$

$\mathfrak{S}(r, \theta, \phi)$ Spherical coordinates.

Example: $\mathfrak{S}(r, \theta, \phi) = r \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

12.1.3 Identity and Selection Matrices

$\mathfrak{1}$ 15×15 identity matrix.

$\mathfrak{A}$ 15×3-matrix to write a 3-vector into all 3-vectors of a 15-vector.

Example: $\mathfrak{A} \cdot \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \end{pmatrix}^T$

$\mathfrak{i}(g)$ 3×15-matrix to extract the g-th 3-vector from a 15-vector.

Example (take the second 3-vector): $\mathfrak{i}(2) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix}_{15}$

$\mathfrak{I}(g)$ 15×3-matrix to insert a 3-vector at g-th position into a 15-vector.

Example (write into first 3-vector): $\mathfrak{I}(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}^T$

$\mathfrak{O}(g)$ 15×15-matrix to return the g-th 3-vector in a 15-vector.

Example (return only entries of vector 2 unequal to 0): $\mathfrak{O}(2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}$

12.1.4 Rotations

$\mathfrak{c}$ Vector of the cross product matrices for three dimensions.

Example: $\mathfrak{c} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^T$

$\mathfrak{C}(\mathrm{v})$ Cross product matrix for a vector.

Example: $\mathfrak{C}(\begin{pmatrix} x & y & z \end{pmatrix}^T) = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$

$\mathfrak{E}(\alpha, \beta, \gamma)$ Euler matrix for rotation by α around the z-axis, then by β around the rotated y-axis and finally by γ around the rotated x-axis. The pole is at $\beta = \pm\pi/2$; integrations over all orientations have to be done with $\cos(\beta)$ and normalization of $1/(8\pi^2)$.

Example (rotate around y- and then the rotated x-axis):

$$\mathfrak{E}(0, \beta, \gamma) = \begin{pmatrix} \cos(\beta) & \sin(\beta)\sin(\gamma) & \cos(\gamma)\sin(\beta) \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ -\sin(\beta) & \cos(\beta)\sin(\gamma) & \cos(\beta)\cos(\gamma) \end{pmatrix}$$

Example (integrate function $f(\alpha, \beta, \gamma)$ over all spatial orientations with normalization):

$\frac{1}{8\pi^2} \int_{-\pi}^{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\pi}^{\pi} \cos(\beta) f(\alpha, \beta, \gamma)\, d\gamma\, d\beta\, d\alpha$

$\mathfrak{R}(v)$ Rotation around the vector v using the angle $|v|$.

Example (rotate by α around $\begin{pmatrix}1\\1\\0\end{pmatrix}$): $\mathfrak{R}(\alpha \begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\\0\end{pmatrix}) = \begin{pmatrix} \cos^2\left(\frac{\alpha}{2}\right) & \sin^2\left(\frac{\alpha}{2}\right) & \frac{\sin(\alpha)}{\sqrt{2}} \\ \sin^2\left(\frac{\alpha}{2}\right) & \cos^2\left(\frac{\alpha}{2}\right) & -\frac{\sin(\alpha)}{\sqrt{2}} \\ -\frac{\sin(\alpha)}{\sqrt{2}} & \frac{\sin(\alpha)}{\sqrt{2}} & \cos(\alpha) \end{pmatrix}$

$\mathfrak{R}(x,y,z)$ Rotation around the vector $v = \begin{pmatrix} x & y & z \end{pmatrix}^T$ using the angle $|v|$.
See also $\mathfrak{R}(v)$

12.1.5 Graph Transformations

$\mathfrak{r}(\ldots)$ Rotate a graph using

a) Euler angles

Parameters are $\{p, \alpha T, \beta T, \gamma T, \alpha A, \beta A, \gamma A, \alpha B, \beta B, \gamma B\}$, or when ignoring attached vertices $\{p, \alpha T, \beta T, \gamma T\}$. p is the particle number, α, β and γ are the Euler angles like in $\mathfrak{E}(\alpha, \beta, \gamma)$. T is the triangle, A the first attached vertex (lower index) and B the second attached vertex (higher index).

Example (rotate in particle no. 5 the first attached vertex in the x/y-plane around it's parent vertex):

$$\mathfrak{r}(5,0,0,0,\alpha A,0,0,0,0,0) = \begin{pmatrix} \cos(\alpha\mathrm{A}) & -\sin(\alpha\mathrm{A}) & 0 & 1-\cos(\alpha\mathrm{A}) & \sin(\alpha\mathrm{A}) & 0 & 0 & 0 & 0 & \\ \sin(\alpha\mathrm{A}) & \cos(\alpha\mathrm{A}) & 0 & -\sin(\alpha\mathrm{A}) & 1-\cos(\alpha\mathrm{A}) & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \\ & & & & & & & & & \ddots \end{pmatrix}$$

b) rotation vector

Parameters are $\{p, vT, vA, vB\}$, or when ignoring attached vertices $\{p, \mathrm{vT}\}$. p is the particle number, vT, vA and vB are rotation vectors like in $\mathfrak{R}(v)$. T is the triangle, A the first attached vertex (lower index) and B the second attached vertex (higher index).

$\mathfrak{t}(\ldots)$ Translate a graph in direction of

a) 3-vector $v = \begin{pmatrix} x & y & z \end{pmatrix}^T$ using $\mathfrak{t}(v)$

b) coordinates x, y and z using $\mathfrak{t}(x,y,z)$

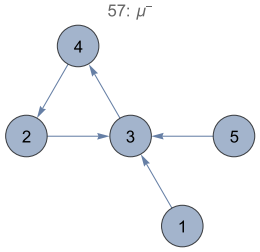
Example: $\mathfrak{t}(x,y,z) = \begin{pmatrix} x & y & z & x & y & z & x & y & z & x & y & z & x & y & z \end{pmatrix}^T$

12.2 General Graph Functions (upper case double stroke letters)

These functions operate on the structure of graphs returning sets of as well as individual vertex indices.

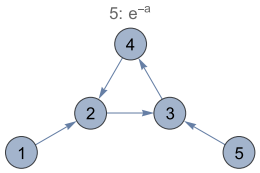
$\mathbb{A}(p)$ Set of vertex indices attached to the triangle (all vertices without the triangle's.), sorted ascending.

Example: $\mathbb{A}(57) = \{1, 5\}$



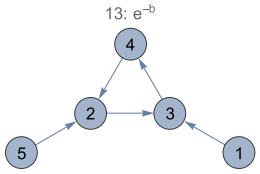
$\mathbb{D}(p, g)$ Determines for vertex g the vertex index in direction of the graph's edges. Select vertex g follow the arrow and read the result.

Examples: $\mathbb{D}(5, 1) = 2$, $\mathbb{D}(5, 3) = 4$



$\mathbb{E}(p)$ Returns the edges of graph p. Every list entry contains the source and destination vertex index. The edge from vertex 6 is included despite it has no physical meaning; in the following example vertex 6 is linked to vertex 2 (the one with the lowest index in the triangle).

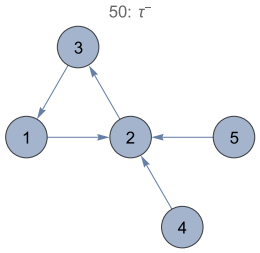
Example: $\mathbb{E}(13) = \{1 \to 3, 2 \to 3, 3 \to 4, 4 \to 2, 5 \to 2, 6 \to 2\}$



(Plots don't include the edge from vertex 6.)

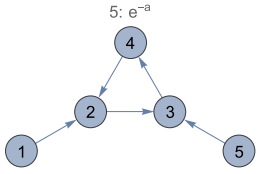
$\mathbb{L}(p, g)$ Returns a set of vertex indices pointing towards g (includes all recursively linked) but does not include triangle indices. Since there are probably no graphs of relevance with recursively linked vertices, only vertices attached to triangle vertices are returned and of relevance.

Examples. $\mathbb{L}(50, 2) = \{4, 5\}$; $\mathbb{L}(50, 3) = \{\,\}$



$\mathbb{N}(p, g)$ Determines the next vertex in the triangle.

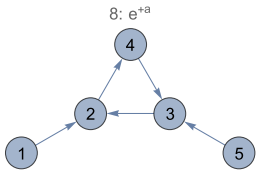
Example: $\mathbb{N}(5, 4) = 2$



See also $\mathbb{P}(p, g)$

$\mathbb{O}(p)$ Returns the triangles' vertices' indices in ascending order.

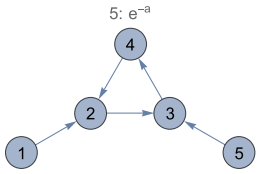
Example: $\mathbb{O}(8) = \{2, 3, 4\}$



See also $\mathbb{T}(p)$

$\mathbb{P}(p, g)$ Determines the prior vertex in the triangle.

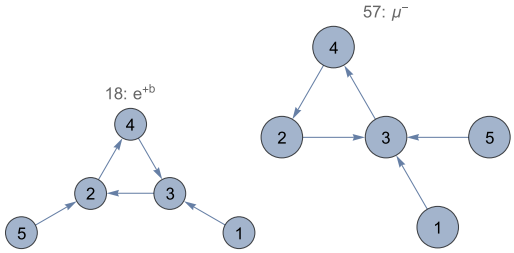
Example: $\mathbb{P}(5, 4) = 3$



See also $\mathbb{N}(p, g)$

$\mathbb{R}(p)$ Get the indices of the rotating vertices of particle p as a set. These are the vertices of the triangle having no attached vertices.

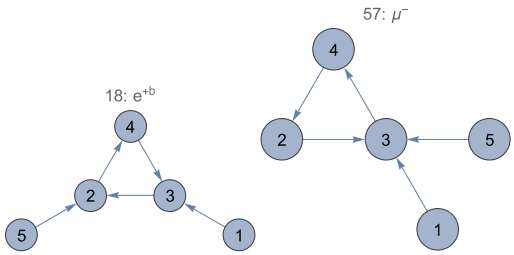
Examples: $\mathbb{R}(18) = \{4\}$; $\mathbb{R}(57) = \{4, 2\}$



See also $\mathbb{S}(p)$

$\mathbb{S}(p)$ Get the non-rotating indices of the triangle as a set. These are the vertices of the triangle having attached vertices.

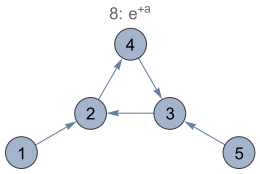
Examples: $\mathbb{S}(18) = \{3, 2\}$; $\mathbb{S}(57) = \{3\}$



See also $\mathbb{R}(p)$

$\mathbb{T}(p)$ Returns the triangles' vertices' indices in the order of edges' direction, beginning with the lowest index.

Example: $\mathbb{T}(8) = \{2, 4, 3\}$

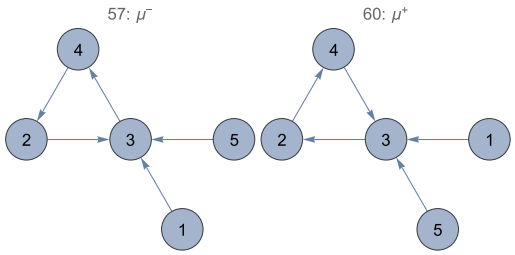


See also $\mathbb{O}(p)$

12.3 Particle Functions (script letters)

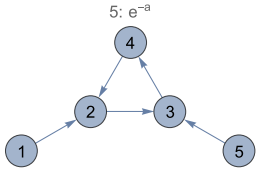
$a(p)$ Get the particle number of p's antiparticle. The difference between particles and antiparticles is the direction of the triangle's edges.

Example. $a(57) = 60$; $a(60) = 57$



$\mathcal{A}(p)$ Returns the 5×5 adjacency matrix of particle p's graph structure. The sixth column and row again stands for the non-physical edge giving the particle's z-axis.

Example: $\mathcal{A}(5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$



See also $\mathbb{G}_p$

$\mathcal{B}(p, \mathrm{Gs})$ Calculates the particle's basis for particle number p and it's graph-vector Gs (e.g. $\mathbb{F}_p$ or $\mathcal{G}(p)$). The basis is defined by:

x-axis is the direction from the triangle's vertex with the lowest index to the one with the next higher index (in standard orientation $\mathcal{G}$ this is $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$)

y-axis is the direction perpendicular to the x-axis within the triangle's plane on the side of the third triangle vertex (in standard orientation $\mathcal{G}$ this is $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$, or x rotated by $\pi/2$ around the z-axis standing perpendicular to the triangle)

z-axis builds a right-handed basis with the x- and y-axes and stands perpendicular to the triangle (in standard orientation $\mathcal{G}$ this is $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$)

Example: $\mathcal{B}(5, \mathcal{G}(5)) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

If $\mathcal{G}$ or more precisely it's triangle is rotated, e.g. by α around the z-axis the result is as:

$\mathcal{B}(5, \mathfrak{r}(5, \alpha, 0, 0) \cdot \mathcal{G}(5)) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\mathcal{C}$ Count of distinct particles.

Example (should be the correct number): $\mathcal{C} = 66$

$d(p)$ Calculates the length of vector $\mathcal{D}(p)$. This is like the norm function in rotation space.

Example (for particle $D = 26$): $d(26) = 2$

Example (for particle $\mu^- = 57$): $d(57) = 1$

$\mathcal{D}(p)$ Returns a matrix to calculate the vector around which rotation interaction's particle rotation acts.

Example: $\mathcal{D}(5) = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 \end{pmatrix}_{15}$

$\mathcal{F}(p, J, j)$ Calculates the direction of the generator of rotation interaction relative to the z-axis edge at the beginning of the rotation for vertex J and orientation state j.

Example: $\mathcal{F}(5, J, j)$ for $J = 1 \ldots 5$ and $j = 1 \ldots 5$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Only vertex 4 is rotating ($\mathbb{R}(5) = \{4\}$) so only the fourth row has elements $\neq 0$ and rotation takes only place while the triangle's vertices are undergoing reflections ($\mathbb{O}(5) = \{2, 3, 4\}$) so only columns 2, 3 and 4 have elements $\neq 0$. Since reflection changes the direction of rotation interaction vectors ($\mathcal{R}(p)$), while the z-axis in particle's current state is static, the values switch between $+1$ and -1.

The rotation generator itself as a vector is calculated by $G.z(p).G$ with $G = \mathcal{G}(p)$ or $G = \mathbb{F}_p$.

See also $z(p)$

$\mathcal{F}(p)$ This is like $\mathcal{F}(p, J, j)$, but for the first rotating vertex in the state before reflections of the triangle's vertices take place.

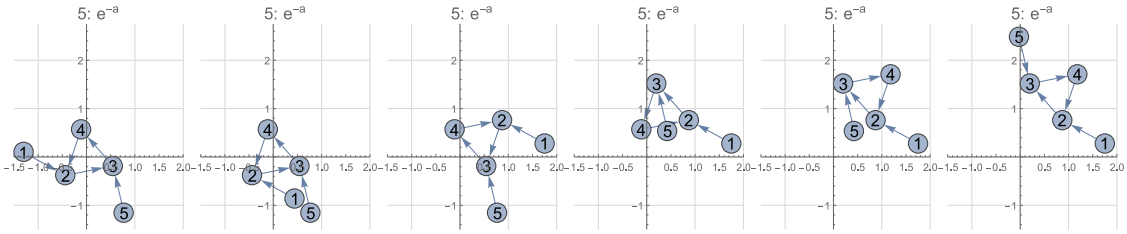
$g(p)$ Returns a vector of five 15×15-matrices each applied to e.g. $\mathcal{G}(p)$ calculating the state of the graph after each reflection (orientation state). Since $\mathcal{O}(p, g)$ only gives the transformation of particle p when reflection of vertex g is done it is useful to have a matrix applying these reflections one after another. Therefore this function is simply shorthand for:

$$\begin{pmatrix} \mathcal{O}(p,1) \\ \mathcal{O}(p,2) \cdot \mathcal{O}(p,1) \\ \mathcal{O}(p,3) \cdot \mathcal{O}(p,2) \cdot \mathcal{O}(p,1) \\ \mathcal{O}(p,4) \cdot \mathcal{O}(p,3) \cdot \mathcal{O}(p,2) \cdot \mathcal{O}(p,1) \\ \mathcal{O}(p,5) \cdot \mathcal{O}(p,4) \cdot \mathcal{O}(p,3) \cdot \mathcal{O}(p,2) \cdot \mathcal{O}(p,1) \end{pmatrix}$$

See also $\mathcal{O}(p, g)$

$g(p, g)$ This function is very similar to $g(p)$. In fact it calls $g(p)$ but retrieves the g-th element from the vector calculated by $g(p)$. In addition it returns the 15×15 identity matrix $\mathbb{1}$ for $g = 0$.

Example: $g(5, g).\mathfrak{t}(5, \ldots).\mathcal{G}(5)$, plotted for $g = 0 \ldots 5$



$\mathcal{G}(p)$ Returns initial coordinates for all 5 vertices = 15 coordinates of particle p. These are as follows: The triangle lies in the x/y-plane centered at $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$. The triangle's edge from the vertex with lowest index to the one with the next higher is parallel to the x-axis. Attached edges point in positive x-direction.

Example: $\mathcal{G}(5) = \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2\sqrt{3}} & 0 & -\frac{1}{2} & -\frac{1}{2\sqrt{3}} & 0 & \frac{1}{2} & -\frac{1}{2\sqrt{3}} & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{2} & -\frac{1}{2\sqrt{3}} & 0 \end{pmatrix}^T$

See also $\mathbb{F}_p$

$i(p)$ Calls $\mathcal{I}(p, 1, 1)$.

See also $\mathcal{I}(p, HT, HT)$

$\mathcal{I}(p)$ Calls $\mathcal{I}(p, h_t, , h_a)$, so with constants corresponding to Planck's constant.

See also $\mathcal{I}(p, HT, HT)$

$\mathcal{I}(p, \mathrm{HT}, \mathrm{HA})$ Returns for particle p a vector of three 15×15-matrices to calculate the effect of interactions on the graph as a transformation generator. Therefore it can be used in a differential equation to calculate time evolution (continuous time and space). It is contracted with the result of the interaction function $i()$ (returns a 3-vector) and applied to the graph (e.g. graph functions $\mathbb{F}_p$) like $i()_p \cdot \mathcal{I}(p) \cdot \mathbb{F}_p$.

Parameters HT and HA define factors of the transformation strength for the triangle resp. the attached vertices.

This function is the infinitesimal version of $\mathcal{M}(p, i(), HT, HT)$ and is correspondingly calculated as the derivatives of the three dimensional interaction function vector $i()$:

$\{\partial_x \mathcal{M}(p,\{x,0,0\},HT,HT), \partial_y \mathcal{M}(p,\{0,y,0\},HT,HT), \partial_z \mathcal{M}(p,\{0,0,z\},HT,HT)\}$ at $x=0$, $y=0$ resp. $z=0$.

Examples for use:

Calculate the dynamics of the muon induced by self interaction without wave terms (shown are only rotation and translation terms): $\partial_t \mathbb{F}_{57} = i(\{\mathbb{F}_{57}\})_{index_{57}=1} \cdot \mathcal{I}(57) \cdot \mathbb{F}_{57} + \frac{h_f \mathfrak{A} \cdot i(\{\mathbb{F}_{57}\})_{index_{57}=1}}{d(57)\mathcal{N}(57)}$

The same for two particles, the muon and anti-muon, including interaction between the two particles $p=\{57,60\}$: $\partial_t \mathbb{F}_p = i(\{\mathbb{F}_{57},\mathbb{F}_{60}\})_{index_p=1 \text{ or } 2} \cdot \mathcal{I}(p) \cdot \mathbb{F}_p + \frac{h_f \mathfrak{A} \cdot i(\{\mathbb{F}_{57},\mathbb{F}_{60}\})_{index_p=1 \text{ or } 2}}{d(index_p)\mathcal{N}(index_p)}$

See also $\mathcal{M}(p,i,HT,HA)$

$\mathcal{K}(p)$ Returns a 15×15-matrix that allows calculation of the graph's contraction point. This is a point of special relevance in the particles' edges length oscillation (see $\mathcal{L}(p,s)$). Expansion and contraction points are depending on the graphs structure at the same coordinates or at different ones. This matrix returns the same point 5 times as a 15-vector.

Example: $\mathcal{K}(50) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}_{15}$

See also $\mathcal{S}(p)$ and for the expansion and contraction process $\mathcal{L}(p,s)$

$\mathcal{L}(p)$ Returns the (maximum) length of all edges of graph/particle p. There are probably no graphs with edge lengths other than $\in \{1,2,3\}$.

Example (for particle $\mu^- = 57$): $\mathcal{L}(57) = 3$

See also $\mathbb{G}_p$

$\mathcal{L}(p,s)$ Every graph's edges lengths are subject to discrete oscillation while creating the particle's properties. This oscillation leads to some dynamics and every time it reaches the maximum length $\mathcal{L}(p)$ interactions take place. This function returns the edges' length in the oscillation state s.

This process of expansion and contraction including the corresponding points $\mathcal{S}(p)$ (for Source) and $\mathcal{K}(p)$ (for sinK) respectively is important since it highly influences particle's self interaction (one of the main parts of classical particle mass).

Since there are only three different $\mathcal{L}(p)$, namely 1, 2 and 3, it's easy to write down the oscillations explicitly:

(Oscillation starts always at maximum length $\mathcal{L}(p)$)

goal: $\mathcal{L}(p)=1$	both direct			
$\mathcal{L}(p,s)$ from $\mathcal{S}$		1		1
back to $\mathcal{K}$		1		1
state s	0		0	
		1		2

goal: $\mathcal{L}(p)=2$	step to goal				both direct				step to goal			
$\mathcal{L}(p,s)$ from $\mathcal{S}$				2		1		1				2
back to $\mathcal{K}$		1		2		1		1		1		2
state s	0		0		0		0		0		0	
		1		2		3		4		5		6

goal: $\mathcal{L}(p)=3$	step to goal						step to goal				both direct				step to goal				step to goal					
$\mathcal{L}(p,s)$ from $\mathcal{S}$						3				2		1		1				2						3
back to $\mathcal{K}$		1		2		3		1		2		1		1		1		2		1		2		3
state s	0		0		0		0		0		0		0		0		0		0		0		0	
		1		2		3		4		5		6		7		8		9		10		11		12

Expansion and contraction points are defined by the order of the vertices (index-numbers). A contracted edge has no influence on their location while expanded ones do. The point of expansion and contraction is always at the tip of directed edges. That means, $\mathcal{K}(p)$ and $\mathcal{S}(p)$ are always located at one of the triangles vertices or if no attached edge is expanded at the triangle's center.

In words for

$\mathcal{L}(p) = 1$ The particle has some coordinates from it's past dynamics, lets say $\mathcal{G}(p)$ or $\mathbb{F}_p$, which always represent the graph with all the edges having maximum length $\mathcal{L}(p)$. Then a contraction to length 0 follows in the order of the vertices indices. First edge 1, then edge 2 and so on; the triangle's edges have to contract and expand at once.

- If vertex 1 is part of the triangle the whole triangle contracts and in this case must contract at one of it's three vertices (that's $\mathcal{K}$) because none of the attached edges can be contracted at that moment. After that the other edges contract in order of their index at their tips.
- If vertex 1 is attached to the triangle it simply contracts to it's edge's tip. As soon as it's the triangles turn it contracts either at it's center (if no attached edges are still expanded) or at it's left vertex with an uncontracted edge attached to it.

As soon as all edges are contracted expansion begins with just the same process: Every edge is expanded in order of their indices; triangle at once. Again to the triangle attached vertices/edges define $\mathcal{S}$, the source point of expansion.

$\mathcal{L}(p) = 2$ Here the process is very similar to above. The only difference is that a length of 2 it not reached in one step. It first has to go to 1, then back to 0 and then to 2.

$\mathcal{L}(p) = 3$ This is the same as above but with three steps - see the table.

See also $\mathcal{K}(p)$, $\mathcal{S}(p)$ and $\mathcal{T}(p)$

$m(p, i)$ Calls $\mathcal{M}(p, i, 1, 1)$.

See also $\mathcal{M}(p, i, HT, HA)$

$\mathcal{M}(p, i)$ Calls $\mathcal{M}(p, i, h_t, , h_a)$, so with constants corresponding to Planck's constant.

See also $\mathcal{M}(p, i, HT, HA)$

$\mathcal{M}(p, i, HT, HA)$ Calculates a 15×15-matrix which defines the effect of the interaction $i()$ (a 3-vector) on a particle. This matrix contains three rotations:

- Rotate the triangle of the graph by applying $\mathfrak{R}(h_t i)$ around the center of the triangle.
- Rotate both attached vertices by $\mathfrak{R}(h_a i)$ around each's parent vertex $\mathbb{D}(p, \text{vertex})$.

Example: Printing an example is not useful since this matrix is even in it's simplest form quite unhandy.

Example for use: In contrast to $\mathcal{I}(p, HT, HA)$ it is used in computations with discrete time and space in regard of waves.

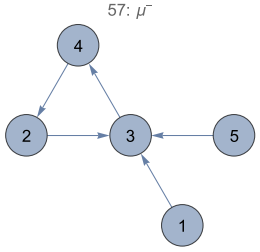
Calculate the dynamics of the muon induced by self interaction without wave terms (shown are only rotation and translation terms): $\Delta\mathbb{F}_{57} = \mathcal{M}(57, i(\{\mathbb{F}_{57}\})_{index_{57}=1}) \cdot \mathbb{F}_{57} + \frac{h_f \mathfrak{A} \cdot i(\{\mathbb{F}_{57}\})_{index_{57}=1}}{d(57)\mathcal{N}(57)}$

The same for two particles, the muon and anti-muon, including interaction between the two $p = \{57, 60\}$: $\Delta\mathbb{F}_p = \mathcal{M}(p, i(\{\mathbb{F}_{57}, \mathbb{F}_{60}\})_{index_p=1 \text{ or } 2}) \cdot \mathbb{F}_p + \frac{h_f \mathfrak{A} \cdot i(\{\mathbb{F}_{57}, \mathbb{F}_{60}\})_{index_p=1 \text{ or } 2}}{d(index_p)\mathcal{N}(index_p)}$

See also $\mathcal{I}(p, HT, HT)$

$\mathcal{N}(p)$ Get the number of rotating vertices of particle p.

Example: $\mathcal{N}(57) = 2$



See also $\mathbb{R}(p)$

$\mathcal{O}(p, g)$ Returns for particle p and every vertex g a 15×15-matrix. Applying these matrices to a graph's 15-vector performs the reflection of vertex g about

a) for attached vertices: it's single parent $\mathbb{D}(p, g)$ or

b) for vertices member of the triangle: about the midpoint of the other two vertices of the triangle.

These matrices only represent the reflection of a single vertex; to calculate the reflections in the correct order, matrices have to be applied in sequence on the graph like e.g.: $\mathcal{O}(p,3)\cdot\mathcal{O}(p,2)\cdot\mathcal{O}(p,1)\cdot\mathcal{G}(p)$

This reflection is the reason for orientation interaction.

Example (for $g=1\ldots5$):

$$\mathcal{O}(5,g)=\{\text{g=1:}\begin{pmatrix}-1&2&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}_{15},\text{g=2:}\begin{pmatrix}1&-2&1&1&0\\0&-1&1&1&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}_{15},$$
$$\text{g=3:}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&1&-1&1&0\\0&0&0&1&0\\0&1&-2&1&1\end{pmatrix}_{15},\text{g=4:}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&1&1&-1&0\\0&0&0&0&1\end{pmatrix}_{15},$$
$$\text{g=5:}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&2&0&-1\end{pmatrix}_{15}\}$$

See also for closer details and a graphical representation $g(p)$ and $g(p,g)$

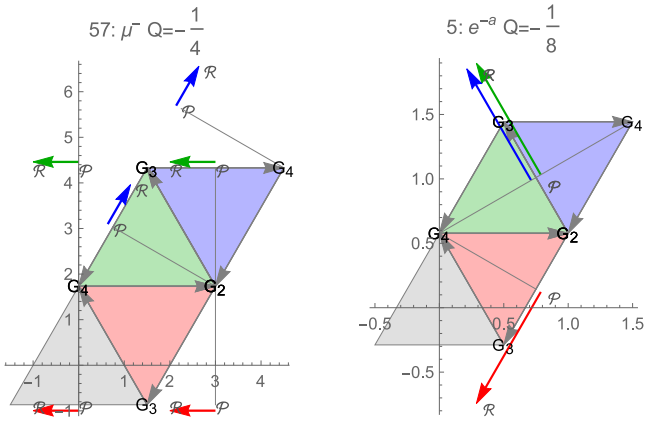
$p(p,g)$ This function returns for every reflection state g a 15×15-matrix to calculate all the rotation points $\mathcal{P}(p)$. In fact it is defined as $\mathcal{P}(p)\cdot g(p,g)$ while $g(p,g)$ uses the reflection matrices $\mathcal{O}(p,g)$.

Example: $p(57,2)=\begin{pmatrix}0&0&0&0&0\\0&-\frac{1}{2}&\frac{3}{2}&0&0\\0&0&0&0&0\\0&\frac{1}{2}&\frac{1}{2}&0&0\\0&0&0&0&0\end{pmatrix}_{15}$

Note that particle $\mu^-=57$ has two rotating vertices and therefore two rotation points $p(p,g)$ and rotation vectors $r(p,g)$.

Examples of use (57: two rotation points, 5: one rotation point; each for three reflection states of the triangle):

$$p(57,2)\cdot\mathcal{G}(57)=\begin{pmatrix}0&0&0&3&-\frac{\sqrt{3}}{2}&0&0&0&0&0&-\frac{\sqrt{3}}{2}&0&0&0&0\end{pmatrix}^T$$
$$p(5,3)\cdot\mathcal{G}(5)=\begin{pmatrix}0&0&0&0&0&0&0&0&0&\frac{3}{4}&\frac{\sqrt{3}}{2}+\frac{1}{4\sqrt{3}}&0&0&0&0\end{pmatrix}^T$$



(These graphics show in red the rotation vectors $\mathcal{R}(p)$ at $\mathcal{P}(p)$ after the first vertex of the triangle has been reflected, in green after the second has been reflected and in blue after the third has been reflected. The gray triangle is the graph's triangle's initial state e.g. $\mathbb{F}_p$ or $\mathcal{G}(p)$.)

See also $r(p,g)$ for the rotation vector.

$\mathcal{P}(p)$ Returns a 15×15-matrix which calculates for every vertex the point it is rotating around. Only vertices of the triangle without attached vertices are subject to rotation. If a vertex does not rotate the result is $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$. This function does not check if the current state of the triangle allows rotation (rotation only takes place during triangle vertices' reflections); it only takes place if the reflection state is a member of $\mathbb{O}(p)$.

There are two kinds of particles in respect to rotation. Please refer to $\mathbb{R}(p)$ and $\mathcal{N}(p)$.

Rotation around the point $\mathcal{P}(p)$ with the rotation vector $\mathcal{R}(p)$ is the reason for rotation interaction.

Example: $\mathcal{P}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}$

See also $p(p, g)$ for calculating the actual coordinates of rotation points in consideration of reflections.

$r(p, g)$ This function takes the same role for$\mathcal{R}(p)$ as $p(p, g)$ does for $\mathcal{P}(p)$. It successively applies reflections $\mathcal{O}(p, g)$ on an initial graph like $\mathbb{F}_p$ or $\mathcal{G}(p)$ and then applies $\mathcal{R}(p)$ to it.

Example: $r(57, 2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}$

See also $p(p, g)$ for the rotation point as well as graphical representations of the calculation results.

$\mathcal{R}(p)$ This function corresponds to $\mathcal{P}(p)$. It returns a 15×15-matrix that allows to calculate the rotation vectors for every vertex if applied to a graph's 15-vector. Every vector belongs to the point calculated by $\mathcal{P}(p)$. Therefore please refer to $\mathcal{P}(p)$. Note that the resulting vectors are always normalized.

Example. $\mathcal{R}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}$

See also $r(p, g)$ for calculating the actual rotation vectors in consideration of reflections.

$\mathcal{S}(p)$ Returns a 15×15-matrix that allows calculation of the graph's expansion point. This is a point of special relevance in the particles' edges length oscillation (see $\mathcal{L}(p, s)$). Expansion and contraction points are depending on the graphs structure at the same coordinates or at different ones. This matrix returns the same point 5 times as a 15-vector.

Example: $\mathcal{S}(50) = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}_{15}$

See also $\mathcal{K}(p)$ and for the expansion and contraction process $\mathcal{L}(p, s)$

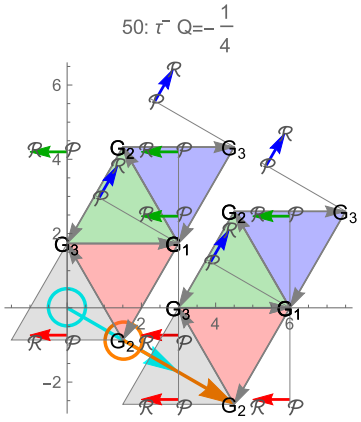
$t(p)$ Returns the number of translation positions, i.e. 1 or 2. If $\mathcal{S}(p) = \mathcal{K}(p)$ there's obviously no translation while edge's lengths are oscillating. If they are different there are exactly two different positions. For details please refer to $\mathcal{T}(p)$.

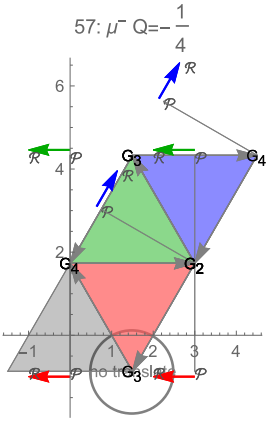
$\mathcal{T}(p)$ According to edge's length oscillation it comes to a translation in the course of build up of particle's properties. While it's possible to define a function following the rules it's easier to analyze the function $\mathcal{L}(p, s)$ in connection with $\mathcal{S}(p)$ and $\mathcal{K}(p)$ as well as the particle's graph structure (in particular the vertice's indices). That leads to a specific translation for every particle. This is representable by a vector from $\mathcal{S}(p)$ to $\mathcal{K}(p)$ multiplied by some quantity (possible values are: 0, 2, 3 and 4) in dependence of the graph's structure and edge's maximum length. This function returns a 15×15-matrix which, applied to a graph, yields a translation vector for the whole graph.

If there's some translation the graphs dynamics at that other location has to be taken into account while calculating interactions within the particle. But this is not visible to other particles since this action takes place only within the particles "search" for it's states.

Example (τ^- shows translation): $\mathcal{T}(50) = \begin{pmatrix} -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 \end{pmatrix}_{15}$

Example (μ^- does not show translation): $\mathcal{T}(57) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15}$





See also $\mathcal{L}(p, s)$ for a detailed description of particle's oscillation

$v(p, w, \mathrm{Gs})$ Calculates the rotation vector for transforming graphs into all their orientations according to the winding of rotation space. There is either only a single v giving the identity if there's no winding in the particle's rotation space, or there are 4 v for all other particles; so w runs from 1 to 1 or from 1 to 4. Gs is the graph's 15-vector.

For parameter $w \in \{1 \ldots 3\}$ this vector is always one that produces an identity matrix, since $w = 1$ is the untransformed graph, and $w \in \{1, 2\}$ are spherical rotations, which allow no winding and opposite orientation of the graph. For $w = 4$ the particle's z-axis is returned as the non-spherical dimension of rotation space.

Example: $v(26, 4, \mathcal{G}(26)) = \begin{pmatrix} 0 & 0 & \pi \end{pmatrix}^T$

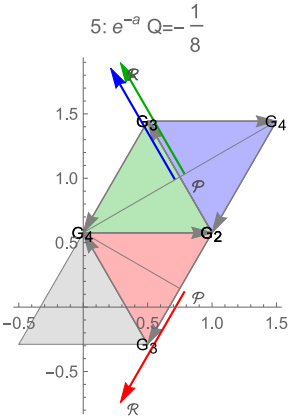
See also $w(p)$ and $\mathcal{W}(p, w)$

$\mathcal{V}(p)$ Return a 15×15-matrix to calculate the edges as vectors.

Example: $\mathcal{V}(5) = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}_{15}$

Example for use:

$\mathcal{V}(5).\mathcal{G}(5) = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 1 & 0 & 0 \end{pmatrix}^T$



$w(p)$ Determines whether rotation space is winded or not. If it's not this function returns 1 because only the identity of the particle's graph is taken into account in interactions. If rotation space is winded, there are 4 instances of the particle's graph; one for the identity and each one for every single dimension in rotation space (3).

Example: $w(26) = 4$

See as $v(p, w, Gs)$ and $\mathcal{W}(p, w)$

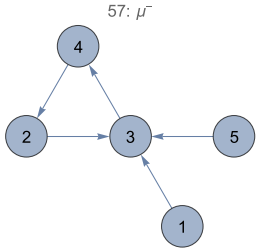
$\mathcal{W}(p, w)$ Returns the rotation matrix for particle p and rotation vector $v(p, w, \mathcal{G}(p))$ in the particle's standard orientation.

This function simply calls $\mathfrak{r}(p, v(p, w, \mathcal{G}(p)))$.

See as $v(p, w, Gs)$ and $\mathfrak{t}(p, vT)$

$\mathcal{X}(p)$ Returns a 3×15-matrix to calculate the center of the graph's triangle by applying it to a graph.

Example: $\mathcal{X}(57) = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}_{15}$



$x(p)$ Returns for particle p a 3×15-matrix to calculate the particle's x-axis.

Example for use: $x(p) \cdot \mathcal{G}(p)$

$y(p)$ Returns for particle p a 3×15-matrix to calculate the particle's y-axis.

Example for use: $y(p) \cdot \mathcal{G}(p)$

$z(p)$ Returns for particle p a list of three 3×15-matrices to calculate the particle's z-axis.

Example for use: $\mathcal{G}(p) \cdot z(p) \cdot \mathcal{G}(p)$

12.4 Convenience Functions (standard letters)

$G()$ Returns the position (3-vector) of a vertex specified by the following parameters:

p	particle number; one of $1 \dots \mathcal{C}$
jW	winding index; 1 is identity, $2 \dots 4$ opposite orientation in rotation space's three dimensions
jT	specifies whether the queried point shall be the one in the base position or the one translated by edge's length's oscillation (see $\mathcal{L}(p, s)$); in the first case $jT = 1$ in the latter $jT = 2$
J	vertex index within the graph; $1 \dots 5$
j	orientation/reflection state; $1 \dots 5$ or if rotation is of interest $\mathbb{R}(p)$
[G]	the graph's 15-vector to operate on, i.e. $\mathbb{F}_p$; if this parameter is not given it defaults to $\mathcal{G}(p)$

$M()$ Returns the point (3-vector) of the reflections center/mirror-point for the reflection defined by:

p	particle number; one of $1 \dots \mathcal{C}$
jW	winding index; 1 is identity, $2 \dots 4$ opposite orientation in rotation space's three dimensions
jT	specifies whether the queried point shall be the one in the base position or the one translated by edge's length's oscillation (see $\mathcal{L}(p, s)$); in the first case $jT = 1$ in the latter $jT = 2$
J	vertex index within the graph whose reflection point is wanted; $1 \dots 5$
j	orientation/reflection state; $1 \dots 5$; if $J \neq j$ no reflection takes place the result is the coordinate of the vertex

[G] the graph's 15-vector to operate on, i.e. $\mathbb{F}_p$; if this parameter is not given it defaults to $\mathcal{G}(p)$

$P(), R()$ Returns the point resp. rotation vector (3-vectors) vertices are rotating around.

p particle number; one of $1 \dots \mathcal{C}$

jW winding index; 1 is identity, $2 \dots 4$ opposite orientation in rotation space's three dimensions

[jT] [only applicable for $\mathcal{P}()$ since $\mathcal{R}()$ is the same for all jT] specifies whether the queried point shall be the one in the base position or the one translated by edge's length's oscillation (see $\mathcal{L}(p, s)$); in the first case $jT = 1$ in the latter $jT = 2$

J vertex index within the graph whose rotation point/vector is wanted; values can only be $\mathbb{R}(p)$ since only these vertices rotate

j orientation/reflection state; only triangle's reflection states $\mathbb{O}(p)$ since only in these states rotations take place

[G] the graph's 15-vector to operate on, i.e. $\mathbb{F}_p$; if this parameter is not given it defaults to $\mathcal{G}(p)$